Properties of changing orientation homeomorphisms of the disk

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Denote by D^2 2-dimensional disk. We will call a cell complex regularly embedded in D^2 if it is a subcomplex of a triangulation of D^2.

Theorem 1. *Let K be a finite connected one-dimensional cell complex regularly embedded in the interior of D^2 and K^0 be the set of all vertices of K. Suppose there exists a homeomorphism $h\colon D^2 \to D^2$ reversing the orientation of D^2 such that $h(K) = K$ and $h(K^0) = K^0$. Then h^2 preserves every vertex of K fixed and leaves every edge (open 1-cell) of K invariant with preserved orientation.*

Denote by $\mathcal{D}(D^2)$ the group of C^∞-diffeomorphisms of D^2. There is a natural right action of the group $\mathcal{D}(D^2)$ on the space of smooth functions $C^\infty(D^2, \mathbb{R})$ defined by the following rule: $(h, f) \longmapsto f \circ h$, where $h \in \mathcal{D}(D^2)$, $f \in C^\infty(D^2, \mathbb{R})$.

Thus, the *stabilizer* of f with respect to the action

$$\mathcal{S}(f) = \{h \in \mathcal{D}(D^2) \mid f \circ h = f\}$$

consists of f-preserving diffeomorphisms of D^2.

Endow the space $\mathcal{D}(D^2)$ with Whitney C^∞-topology and its subspace $\mathcal{S}(f)$ with the induced one. Denote by $\mathcal{S}_{\mathrm{id}}(f)$ the identity path component of $\mathcal{S}(f)$.

Theorem 2. *Let $f\colon D^2 \to \mathbb{R}$ be a Morse function. Suppose there exists $h \in \mathcal{S}(f)$ changing the orientation of D^2. Then exists diffeomorphism $g \in \mathcal{S}(f)$ such that*

- *$g = h$ on a neighborhood of ∂D^2,*
- *$g^2 \in \mathcal{S}_{\mathrm{id}}(f)$.*

References

[1] S. I. Maksymenko, *Homotopy types of stabilizers and orbits of Morse functions on surfaces*, *Ann. Global Anal. Geom.* **29** (2006), no. 3, 241–285. MR MR2248072 (2007k:57067)