Coexistence of Homoclinic Trajectories

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The nonperiodic trajectory of discrete dynamical systems is called n-homoclinic, if its α- and ω-limit sets coincide and are the same cycle of period n. For each point x of this cycle we will call the subsequence of points of homoclinic trajectory which tends to it as x-subsequence.

We say that the homoclinic trajectory is one-sided, if for every point x of this cycle every its x-subsequence tends to it from one side. If at least for one point x of this cycle its x-subsequence tends to it from both sides, we will call the trajectory as two-sided homoclinic trajectory.

Theorem 1. *The ordering* $1 \triangleright 3 \triangleright 5 \triangleright 7 \triangleright \ldots \triangleright 2 \cdot 1 \triangleright 2 \cdot 3 \triangleright 2 \cdot 5 \triangleright \ldots \triangleright 2^2 \cdot 1 \triangleright 2^2 \cdot 3 \triangleright 2^2 \cdot 5 \triangleright \ldots$ *represents the coexistence of homoclinic trajectories of one-dimensional systems in the following sence.*

If one-dimensional dynamical system has a one-sided n-homoclinic trajectory or two-sided $n/2$-homoclinic trajectory, then it also has a one-sided m-homoclinic trajectory or two-sided $m/2$-homoclinic trajectory for each $m \triangleleft n$.

It has been proven the following theorem.

Theorem 2. *Every one-dimensional dynamical system that has a cycle of period $n \neq 2^i$ will also have a one-sided n-homoclinic trajectory.*

References

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