Unique solvability of the nonlocal problem with integral condition for nonhomogeneous differential equations of second order

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Let $H(([T_1, T_2] \cup [T_3, T_4]) \times \mathbb{R}_+)$ be a class of entirie function, $K_{L,M}$ be a class of quasi-polynomials of the form $f(t,x) = \sum_{i=1}^{n} \sum_{j=1}^{m} Q_{ij}(t,x) \exp[\alpha_i x + \beta_j t]$, where $Q_{ij}(t,x)$ are given polynomials, $L \subseteq \mathbb{C}$, $M \subseteq \mathbb{C}$ $\alpha_i \in L$, $\alpha_k \neq \alpha_l$, for $k \neq l$, $\beta_j \in M$, $\beta_k \neq \beta_l$, for $k \neq l$.

Each quasi-polynomial defines a differential operator $f\left(\frac{\partial}{\partial \lambda}, \frac{\partial}{\partial \nu}\right)$ of finite order on the class of certain function, in the form

$$\sum_{i=1}^{m} \sum_{j=1}^{m} Q_{ji}\left(\frac{\partial}{\partial \lambda}, \frac{\partial}{\partial \nu}\right) \exp\left[\alpha_i \frac{\partial}{\partial \lambda} + \beta_j \frac{\partial}{\partial \nu}\right] \Bigg|_{\lambda=0,\nu=0}.$$

In the strip $\Omega = \{(t,x) \in \mathbb{R}^2 : t \in (T_1, T_2) \cup (T_1, T_2), x \in \mathbb{R}_+\}$ we consider of the problem with integral conditios

$$\left[\frac{\partial^2}{\partial t^2} + a\left(\frac{\partial}{\partial x}\right)\frac{\partial}{\partial t} + b\left(\frac{\partial}{\partial x}\right)\right] U(t,x) = f(t,x), \tag{1}$$

satysfies nonlocal-integral conditions

$$\int_{T_1}^{T_2} U(t,x)dt + \int_{T_3}^{T_4} U(t,x)dt = 0; \quad t \in [T_1, T_2] \cup [T_3, T_4]), \tag{2}$$

$$\int_{T_1}^{T_2} tU(t,x)dt + \int_{T_3}^{T_4} tU(t,x)dt = 0; \tag{3}$$

where $a\left(\frac{\partial}{\partial x}\right)$, $b\left(\frac{\partial}{\partial x}\right)$ are differential expressions with entire functions $a(\lambda), b(\lambda) \neq const$.

Solution of the problem (1), (2), (3) according to the differential-symbol method [1] is represented in the form

$$U(t,x) = f\left(\frac{\partial}{\partial \nu}, \frac{\partial}{\partial \lambda}\right) \Big\{G(t,\nu,\lambda) \exp[\lambda x]\Big\} \Bigg|_{\lambda=\nu=0},$$

where $G(t,\nu,\lambda)$ is a solution of the problem:

$$\left[\frac{d^2}{dt^2} + a(\lambda)\frac{d}{dt} + b(\lambda)\right] G(t,\nu,\lambda) = \exp[\nu t],$$

$$\int_{T_1}^{T_2} G(t,\nu,\lambda) + \int_{T_3}^{T_4} G(t,\nu,\lambda) = 0,$$

$$\int_{T_1}^{T_2} tG(t,\nu,\lambda) + \int_{T_3}^{T_4} tG(t,\nu,\lambda) = 0.$$

This problem is a continuos works [2, 3].

References

[1] P. I. Kalenyuk, Z. M. Nytrebych, Generalized Scheme of Separation of Variables. Differential-Symbol Method. Publishing House of Lviv Polytechnic National University, 2002. 292Z. p. (in Ukrainian).

[2] P. I. Kalenyuk, Z. M. Nytrebych, I .V .Kohut, G. Kuduk, Problem for nonhomogeneous second order evolution equation with homogeneous integral conditions, Math. Methods and Phys.- Mech. Polia. Vol. 58, no. 1. P. 7–19(2015).

[3] P. I. Kalenyuk, G. Kuduk, I .V .Kohut, and Z. M. Nytrebych, Problem with integral conditiond for differential-operator equation. J. Math. Sci. Vol. 208, No. 3, 267–276(2015).