Infinitesimal transformations of a symmetric Riemannian space of the first class

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P. A. Shirokov ([1]) found all irreducible symmetric Riemannian spaces V_n of the first class. For $n = 4$, the metric tensor $g_{ij}(x)$ of such spaces in the Riemannian coordinate system with origin at the point M_0 $(x^h = 0)$ has the following form:

$$g_{ij}(x) = \underset{\circ}{g_{ij}} + \frac{1}{3}\left(b_{i\alpha}b_{j\beta} - b_{ij}b_{\alpha\beta}\right) x^\alpha x^\beta, \tag{1}$$

where

$$\left(\underset{\circ}{g_{ij}}\right) = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad (b_{ij}) = \begin{pmatrix} \xi_1 & 0 & 1 & 0 \\ 0 & \xi_2 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \tag{2}$$
$$(\xi_i = \pm 1, \ i = 1, 2)$$

In (2) $\underset{\circ}{g_{ij}}$ and b_{ij}, in the terminology of P. Shirokov, are values of the components of the 1st and 2nd fundamental tensors at the origin of the Riemannian coordinates.

For an arbitrary Riemannian space $V_n(x; g(x))$ S. M. Pokas' ([2]) introduced the concept of the second approximation space $\widetilde{V}_n^2(y; \widetilde{g}(y))$:

$$\widetilde{g}_{ij}(y) = \underset{\circ}{g_{ij}} + \frac{1}{3} \underset{\circ}{R_{i\alpha\beta j}}\, y^\alpha y^\beta \tag{3}$$

where $\underset{\circ}{g_{ij}} = g_{ij}(M_0), \underset{\circ}{R_{i\alpha\beta j}} = R_{i\alpha\beta j}(M_0),$ *and* $M_0 \in V_n$ *is an arbitrary point of source space.*

A comparison of (3) and (1) leads to the conclusion that the symmetric Riemannian space of the first class V_n is isometric to the space of the second approximation $\widetilde{V}_n^2$. Consequently, the Lie group of infinitesimal transformations $\widetilde{G}_r$ of the space $\widetilde{V}_n^2$ is isomorphic to the Lie group of infinitesimal transformations G_r of a symmetric Riemannian space of the first class V_n.

Using the results of studies of infinitesimal transformations of the second approximation space $\widetilde{V}_n^2$, we obtain statements.

Theorem 1. *The infinitesimal conformal transformation of the second degree of the symmetric Riemannian space of the first class V_n is necessarily homothetic.*

Theorem 2. *In the symmetric Riemannian space of the first class V_n, a Lie group of motions G_8 exists.*

The basis and structure of this group are found.

References

[1] P. A. Shirokov. Selected Geomeric Works. *Kazan University Press*, 389–400, 1966.
[2] S. M. Pokas'. *Infinitesimal conformal transformations in the Riemannian space of the second approximation*, volume 7 of *Proc. of the Intern. Geom. Center*, 36–50, 2014.