Automorphisms of the Kronrod-Reeb graphs of Morse functions on 2-sphere

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Let M be a compact two-dimensional manifold and , $f \in C^{\infty}(M,\mathbb{R})$ is Morse function and Γ_f its Kronrod-Reeb's graph. We denote the $O(f) = \{f \circ h \mid h \in D(M)\}$ orbit of f with respect to the natural right action of the group of diffeomorphisms $D(M)$ on $C^{\infty}(M,\mathbb{R})$, and $S(f) = \{h \in D(M) \mid f \circ h = f\}$ is the stabilizer of this function. It is easy to show that each $h \in S(f)$ induces an automorphism of the graph Γ_f. Let also $S'(f) = S(f) \cap D_{\mathrm{id}}(M)$ be a subgroup of $D_{\mathrm{id}}(M)$, consisting of diffeomorphisms preserving f and isotopic to identical mappings and G_f be the group of automorphisms of the Kronrod-Reeb graph induced by diffeomorphisms belonging to $S'(f)$. This group is the key ingredient for calculating the homotopy type of the orbit $O(f)$.

In the previous article, the authors describe the structure of groups G_f for Morse functions on all orientational surfaces, except for sphere and torus. In this paper we study the case $M = S^2$. In this situation Γ_f is always a tree, and therefore all elements of the group G_f have a common fixed $\mathrm{Fix(G_f)}$ subtree, which can be even from one point. The main result is to calculate the groups G_f for all Morse functions $f : S^2 \to \mathbb{R}$ in which $\mathrm{Fix(G_f)}$ is not the point.

Theorem 1. *Let $f \in C^{\infty}(S^2,\mathbb{R})$ be Morse function on a sphere. Suppose that all elements of the group G_f have a common fixed edge E. Let $x \in E$ be an arbitrary point and A and B is the closure of the connected components $S^2 \setminus p^{-1}(x)$. Then A and B-double discs are invariant with respect to $S'(f)$, the restriction of $f|_A, f|_B$ are Morse functions and we have the following isomorphism:*

$$\phi : G_f \to G_{f_{|A}} \times G_{f_{|B}},$$

is determined by the formula $\phi(\gamma) = (\gamma_{|\Gamma_A};\ \gamma_{|\Gamma_B})$.

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