Diffeomorphisms groups of certain singular foliations on lens spaces

Sergiy Maksymenko
(Institute of Mathematics of NAS of Ukraine, Tereshchenkivska str.,3, Kyiv, Ukraine)
E-mail: maks@imath.kiev.ua

Olexandra Khokhliuk
(Taras Shevchenko National University of Kyiv, 64/13, Volodymyrska St., Kyiv, Ukraine)
E-mail: khokhliyk@gmail.com

Let

$$T = D^2 \times S^1 = \{(x, y, w) \in \mathbb{R}^2 \times \mathbb{C} \mid x^2 + y^2 \leq 1,\ |w| = 1\}$$

be a solid torus, $C_r = \{z \in D^2 \mid |z| = r\} \subset D^2$, $r \in [0, 1]$ and

$$\mathcal{F}_T = \{C_r \times S^1\}_{r\in[0,1]}$$

be a foliation on T into 2-tori parallel to the boundary and one singular circle $C_0 \times S^1$, which is the central circle of the torus T.

Denote by $\mathcal{D}(\mathcal{F}_\mathcal{T}, \partial T)$ the group of leaf preserving diffeomorphisms of T, which are fixed on ∂T.

Theorem 1. *The group $\mathcal{D}(\mathcal{F}_T, \partial T)$ is contractible.*

Let $L_{p,q}$ be a lens space, that is, a three-manifold obtained by gluing two solid tori $T_1 = T_2 = D^2 \times S^1$ via some diffeomorphism $\phi : \partial T_1 \to \partial T_2$ of their boundaries.

The topological type of the lens space is determined by the isotopy class of the image of the meridian of ∂T_1 in ∂T_2.

Notice that each of the solid tori T_i, $i = 1, 2$, has a foliation $\mathcal{F}_i$ on two-dimensional tori

$$\mathcal{F}_{T_i} = \{C_r \times S^1\}_{r\in[0,1]}$$

with one singular leaf $C_0 \times S^1$ being the central circle of the torus T_i.

Since ∂T_i is a leaf of foliation $\mathcal{F}_{Ti}$, we see that the foliations on $\mathcal{F}_{T_1}$ and $\mathcal{F}_{T_2}$ determine a certain foliation $\mathcal{F}_{p,q} = \mathcal{F}_1 \cup \mathcal{F}_2$ on $L_{p,q}$ into 2-tori parallel to the boundary and two singular circles being the central circles C_1 and C_2 of the solid tori T_1 and T_2.

Denote by $\mathcal{D}(\mathcal{F}_{p,q})$ the group of leaf preserving diffeomorphisms of $L_{p,q}$ which also preserve their orientations .

Theorem 2. *The group $\mathcal{D}(\mathcal{F}_{p,q})$ is homotopy equivalent to $\mathbb{Z} \times S^1 \times S^1$ for $L_{p,q} = S^2 \times S^1$ and to $S^1 \times S^1$ in all other cases.*