Exact Morse functions on Kendall shape spaces

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We introduce and investigate several Morse functions on Kendall shape spaces. The main attention is given to a properly normalized oriented area and Coulomb potential on shape spaces of planar n-gons. It is easy to verify that convex and equiangular shapes of regular n-gons are critical points of the foregoing functions. We prove that, in many cases, these critical points are non-degenerate, and compute their Morse indices. It follows that, for odd n, the normalized oriented area is an exact Morse function on the Kendall shape space of planar n-gons. For even n, we are only able to show that the normalized oriented area has the minimal possible number of critical points equal to the Lusternik-Schnirelmann category of the shape space considered. The proofs use our earlier results on Morse functions on moduli spaces of planar linkages obtained jointly with G.Panina and D.Siersma. For $n = 4, 5$, we also show that any two points in the shape space can be connected by a differentiable curve consisting of equilibrium configurations of the normalized Coulomb potential with varying charges of vertices. Possible analogs and generalizations to spatial shape spaces will also be mentioned.