Properties of some algebras of entire functions of bounded type, generated by a countable set of polynomials on a Banach space

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Let X be a complex Banach space. Let $\mathbb{P} = \{P_1, P_2, \ldots, P_n, \ldots\}$ be a countable set of algebraically independent continuous $n-$homogeneous complex-valued polynomials on X for every positive integer n. Let us denote by $H_{b\mathbb{P}}(X)$ the closed subalgebra, generated by the elements of $\mathbb{P}$, of the Fréchet algebra $H_b(X)$ of all entire functions of bounded type on X.

In this talk we will discuss some properties of the algebra $H_{b\mathbb{P}}(X)$. For instance, we will show that every $f \in H_{b\mathbb{P}}(X)$ can be uniquely represented in the following form

$$f(x) = f(0) + \sum_{n=1}^{\infty} \sum_{k_1+2k_2+\ldots+nk_n=n} a_{k_1,k_2,\ldots,k_n} (P_1(x))^{k_1} (P_2(x))^{k_2} \cdots (P_n(x))^{k_n},$$

where $x \in X$, $a_{k_1,k_2,\ldots,k_n} \in \mathbb{C}$ and $k_1, k_2 \ldots, k_n \in \mathbb{N} \bigcup \{0\}$. Consequently, the spectrum (the set of all continuous linear multiplicative functionals) of the algebra $H_{b\mathbb{P}}(X)$ is in one-to-one correspondence with some set of sequences of complex numbers. We will prove the upper estimate for sequences of this set. We will also describe the spectrum of algebra $H_{b\mathbb{P}}(X)$ in case when X is a closed subspace of the space l_∞ such that X contains the space c_{00} for some special form of the set $\mathbb{P}$.