On the separability of the topology on the set of the formal power series

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Let we have the sequence $(a_n)_{n=0}^{\infty}$ of the elements from any ring K. The expression $a_0 + a_1x + a_2x^2 + ... = \sum_{i=0}^{\infty} a_i x^i$, where $+$ and x are the formal symbols, a_i are coefficients, a_0 – intercept term is called the formal power series.

Sometimes the expression $f(x) = a_0 + a_1x + a_2x^2 + ...$ is called the another form of the representation of the sequence $(a_n)_{n=0}^{\infty}$. It is convenient to consider the finite sequence as the infinite one. The corresponding formal power series is called the polynomial. The set of the formal power series with coefficients from K is denote by $K[[x]]$.

The set $K[[x]]$ is a ring with identity with respect to the addition and the multiplication of the formal power series: $\sum_{i=0}^{\infty} a_i x^i + \sum_{i=0}^{\infty} b_i x^i = \sum_{i=0}^{\infty}(a_i + b_i)x^i$ and $(\sum_{i=0}^{\infty} a_i x^i)(\sum_{i=0}^{\infty} b_i x^i) = \sum_{k=0}^{\infty}(\sum_{i+j=k} a_i b_j)x^k$. The sets $I_k = x^k K[[x]], k = 0, 1, 2, ...$ are the principal ideals of the ring $K[[x]]$. If these ideals are considered as the open sets, then they and the empty set form the topological structure on the $K[[x]]$.

In the topological space $(R[[x]], \tau)$ if the formal power series $f(x)$ has nonzero intercept term, then its neighborhood is $R[[x]]$. If $f(x)$ has no intercept term, then one can consider any I_k as the neighborhood, where k is not greater than the power of the minimal monomial term of the series $f(x)$.

Proposition 1. *The topological space $(R[[x]], \tau)$ is separable.*

In this space the set M of all polynomials with the integer coefficients is denumerable as it is equal to $\bigcup_{n \in N} P_n$, where P_n is the set of all polynomials with the integer coefficients the powers of which are less than or equal to n which is equivalent to Cartesian product $Z \times Z \times \ldots \times Z$, where Z is repeated $n+1$ times. Moreover the closure $\bar{M}$ is equal to $R[x]$.

References

[1] Lando S.A. *Lektsii o proizvodyaschih funktsiyah.* Moskva : MTsNMO, 2007.