Braids, links, strings and algorithmic problems of topology

Nikolaj Glazunov
(NAU, Kiev, Ukraine)
E-mail: glanm@yahoo.com

V.V. Sharko in his book [5] has investigated functions on manifolds. Braids intimately connect with functions on manifolds. These connections are represented by mapping class groups of corresponding discs, by fundamental groups of corresponding punctured discs. and by some other topological or algebraic structures.

Below we follow to [1, 2, 3, 6] and references therein.

Definition 1. (Configuration spaces of the ordered sets of points). Let M be a topological space and let M^n be the product of n spaces M with the topology of the product. Put

$$\mathcal{F}_n(M) = \{(u_1, \ldots, u_n) \in M^n; u_i \neq u_j, i \neq j\}.$$

Remark 2. If M is a topological space of the dimension $dim\ M$ (possibly with the boundary ∂M) then the dimension of $\mathcal{F}_n(M)$ is equal $n \cdot dim\ M$. The topological space $\mathcal{F}_n(M)$ is connected.

Definition 3. The fundamental group $\pi(\mathcal{F}_n(M))$ of the manifold $\mathcal{F}_n(M)$ is called the group of pure braids with n strands.

Let now M be a connected topological manifolds of the dimension ≥ 2, $M^{in} = M \setminus \partial M$, $Q_m \subset M^{in}$, Q_m contains $m \geq 0$ points. Put

$$\mathcal{F}_{m,n}(M) = \mathcal{F}_n(M \setminus Q_m).$$

and for symmetric group S_n put

$$\mathcal{G}_{m,n} = \mathcal{F}_{m,n}(M)/S_n.$$

Definition 4. The fundamental group $\pi(\mathcal{G}_{m,n})$ is called the braids group of the manifold $M \setminus Q_m$ with n strands.

Remark 5. In the case of $M = \mathbb{R}^2$ we obtain groups of pure braids and braids in the sense of E. Artin and A. Markov.

Let M be a three dimensional topological manifold possible with the boundary ∂M. Recall that a geometric link in M is a locally flat closed one dimensional submanifold in M.

A.A. Markov [3] gave the description of the set of isotopic classes of oriented links in $\mathbb{R}^3$ in terms of braids. For manifolds of the dimension grater than 3 A.A. Markov [2] has proved the undecidability of the problem of homeomorphy.

For algorithmic and computer-algebraic investigations of braids, links and strings we have to represent corresponding data structures and algorithms. These data structures and algorithms are constructive mathematical objects in the sense of A. Markov. The processing of these constructive objects require corresponding constructive semantics. A.A. Markov (see [4] and references therein) began to construct the semantics. Follow to ([4] and references therein), and specialize Markov results to braids, links and strings we have

Proposition 6. *Let we have a description of braids, links and strings as constructive mathematical objects of the language L_2. Then any closed formula of the language L_2 which is inferred from the valid formula of the language L_2, is valid.*

We will present the interpretation of this Proposition on examples from [1, 3, 6, 7, 8].

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