Stabilizers of smooth functions on 2-torus

Bohdan Feshchenko
(Institute of Mathematics of NAS of Ukraine)
E-mail: `fb@imath.kiev.ua`

Let M be a smooth compact surface. The group of diffeomorphisms $\mathcal{D}(M)$ naturally acts from the right on the space of smooth functions $C^\infty(M)$ by the following rule: $\gamma : C^\infty(M) \times \mathcal{D}(M) \to C^\infty(M)$, $\gamma(f, h) = f \circ h$. For the given smooth function $f \in C^\infty(M)$ we denote by $\mathcal{S}(f)$ and $\mathcal{O}(f)$ the stabilizer and the orbit of f with respect to the action γ. Endow strong Whitney topologies on $C^\infty(M)$ and $\mathcal{D}(M)$; these topologies induce some topologies on $\mathcal{O}(f)$ and $\mathcal{S}(f)$. We denote by $\mathcal{D}_{\mathrm{id}}(M)$ and $\mathcal{O}_f(f)$ connected components of $\mathcal{D}(M)$ and $\mathcal{O}(f)$ which contain id and f respectively; we also set $\mathcal{S}'(f) = \mathcal{S}(f) \cap \mathcal{D}_{\mathrm{id}}(M)$.

Let Γ_f be a Kronrod-Reeb graph of a smooth function f. It is easy to see that each $h \in \mathcal{S}'(f)$ induces an automorphism $\rho(h)$ of the graph Γ_f, and the correspondence $\rho : \mathcal{S}'(f) \to \mathrm{Aut}(\Gamma_f)$ is a homeomorphism. The image of $\rho(\mathcal{S}'(f))$ in $\mathrm{Aut}(\Gamma_f)$ will be denoted by $G(f)$. More details can be found in [6].

In the series of papers [1]–[5] S. Maksymenko and the author described an algebraic structure of $\pi_1\mathcal{O}_f(f)$ for Morse functions on 2-torus. In my talk I am going to present algebraic structures of groups $\pi_0\mathcal{S}'(f)$ and $G(f)$ for Morse functions on 2-torus.

References

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