Equilibrium positions of nonlinear differential-algebraic systems

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We investigate the problem of constructing solutions $z(t) \in \mathbb{C}^1[a,b]$ of the nonlinear differential algebraic system [1, 2, 3]

$$A(t)z'(t) = B(t)z(t) + f(t) + Z(z,t). \tag{1}$$

Here $A(t),\ B(t) \in \mathbb{C}_{m\times n}[a,b]$ is a continuous matrices, $f(t) \in \mathbb{C}[a,b]$ is a continuous vector. We consider a nonlinear function $Z(z,t)$ that assume twice continuously differentiable by z in a certain region $\Omega \subseteq \mathbb{R}^n$ and continuous in $t \in [a,b]$. We call the equilibrium position of the system (1) a function $z(t) \in \mathbb{C}^1[a,b]$, that satisfies two conditions $A(t)z' = 0,\ B(t)z + f(t) + Z(z,t) = 0$. In the simplest case, under the condition $B(t) \equiv B,\ f(t) \equiv f - \text{const},\ Z(z,t) \equiv Z(z)$, the equilibrium position $z(t) \equiv z - \text{const}$ of a nonlinear differential-algebraic system (1) defines the equation

$$\varphi(z) := B\,z + f + Z(z) = 0. \tag{2}$$

To solve the equation (2), we apply the Newton method [4, 5].

Lemma 1. *Assume that the following conditions are satisfied for the equation (2):*

(1) The nonlinear vector function $\varphi(z)$ in the neighborhood Ω of the point z_0, has the root $z^* \in \mathbb{R}^n$.

(2) In the indicated neighborhood of the zeroth approximation z_0 the inequalities

$$\left|\left|J_k^+\right|\right| \le \sigma_1(k), \left|\left|d^2\varphi(\xi_k\,; z^* - z_k)\right|\right| \le \sigma_2(k)\cdot||z^* - z_k||, \quad \theta := \sup_{k\in N}\left\{\frac{\sigma_1(k)\sigma_2(k)}{2}\right\}$$

are satisfied.

Then under the conditions

$$P_{J_k^*} = 0, \quad J_k := \varphi'(z_k) \in \mathbb{R}^{m\times n}, \quad \theta\cdot|z^* - z_0| < 1$$

an iterative scheme

$$z_{k+1} = z_k - J_k^+\,\varphi(z_k)$$

is applicable to find the solution z^* of the equation (2).

The vector function z^* is the equilibrium position of the differential algebraic system (1).

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