Expand-contract plasticity of unit balls and related results

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The talk deals with results received by author and her scientific advisor V.M. Kadets. These results has become the base for two papers (see [2] and [4]).

A metric space M is called *expand-contract plastic* (or simply, an EC-space) if every non-expansive bijection from M onto itself is an isometry. The EC-plasticity of totally bounded metric spaces and the unit balls of strictly convex Banach spaces has been proved in [3, Theorem 1.1] and [1, Theorem 2.6] respectively. In particular, the unit balls of all finite-dimensional Banach and of all Hilbert spaces are plastic. On the other hand, there are bounded closed convex sets in an infinite-dimensional Hilbert space that are not EC-spaces [1, Example 2.7]. It is an open question whether unit balls of all Banach spaces are EC-spaces. We have answered this question in the positive for concrete space ℓ_1, which is not strictly convex.

Theorem 1. *The unit ball of ℓ_1, is an EC-space.*

There is another problem connected with the previous one. For which pairs (E, M) of Banach spaces every bijective non-expansive map $F\colon B_E \to B_M$ is an isometry? Unfortunately, we are not able to answer this question in full generality. However, the following results in this direction have been received.

Theorem 2. *Let $F\colon B_E \to B_M$ be a bijective non-expansive map. If M is strictly convex, then F is an isometry.*

Theorem 3. *Let $F\colon B_E \to B_{\ell_1}$ be a bijective non-expansive map. Then F is an isometry.*

Theorem 4. *Let M be a finite-dimensional Banach space, $F\colon B_E \to B_M$ be a bijective non-expansive map. Then F is an isometry.*

Rerefences

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[2] V. Kadets, O. Zavarzina. Plasticity of the unit ball of ℓ_1, *Visn. Hark. nac. univ. im. V.N. Karazina, Ser.: Mat. prikl. mat. meh.* 83 : 4–9, 2017.

[3] S.A. Naimpally, Z. Piotrowski, E.J. Wingler. Plasticity in metric spaces. *J. Math. Anal. Appl.*, 313 : 38–48, 2006.

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