A categorical approach to physics beyond the Standard Model

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A modern version of the unified theory of fundamental interactions is the theory of superstrings and D-branes, which combines quantum field theory and the general theory of relativity into a unified field theory [1]. The ten-dimensional space on which the theory of superstrings is defined can be represented as a direct product of 4-dimensional and 6-dimensional spaces:

$$M_{10} = M_4 \times K_6 \ ,$$

where the manifold M_4 is a four-dimensional space-time, and K is the space of extra dimensions. To obtain acceptable solutions consistent with the Standard model $SU(3)\times SU(2)\times U(1)$, restrictions on compactification are introduced [2]. Therefore, we usually deal with Calabi-Yau manifolds or orbifolds as spaces of extra dimensions. To describe such a space, we use the notion of a topological space, X as a set, in which a system of open sets is distinguished with the following properties:

1) the union of any number of open sets is open;
2) the intersection of a finite number of open sets is open.

On the topological space we can introduce the structure of a ring, $\mathcal{O}_X$. The sheaf of rings of smooth functions on the topological space X consists of the following data:

1) on each open set $U \subset X$ the ring of smooth functions $\mathcal{O}(U)$ is defined;
2) a set of restriction maps $r^U_V : \mathcal{O}(U) \to \mathcal{O}(V)$ satisfying the following conditions is defined: $r^U_U = 1_U,\ r^V_W r^U_V = r^U_W$ for $W \subset V \subset U$;
3) for any open covering $U = \bigcup_i U_i$ there exists an exact sequence

$$0 \to \mathcal{O}(U) \to \prod_i \mathcal{O}(U_i) \to \prod_{i,j} \mathcal{O}(U_i \cap U_j).$$

Locally free sheaf is determined as:

$$E_X:\ 0 \to E_X|_U \to \mathcal{O}_X^{\oplus p}|_U \to 0.$$

Coherent sheaf is determined as:

$$B_X:\ 0 \to B_X|_U \to \mathcal{O}_X^{\oplus p_1}|_U \to \mathcal{O}_X^{\oplus p_2}|_U \to \ldots \to \mathcal{O}_X^{\oplus p_i}|_U \to 0.$$

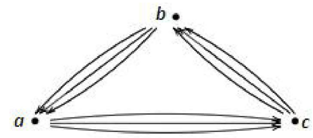
In physics D-branes are associated with coherent sheaves [3]. For physical applications associated with phase transitions from one D-brane to another, the concept of a category is used. In this case we are dealing with a complex of sheaves,

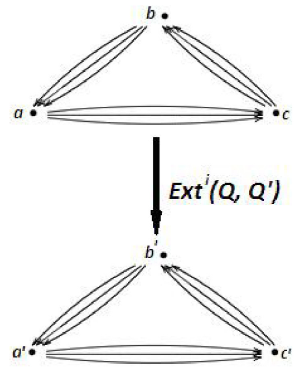
$$B^\bullet : \ldots \stackrel{d^{i-2}}{\to} B^{i-1} \stackrel{d^{i-1}}{\to} B^i \stackrel{d^i}{\to} B^{i+1} \stackrel{d^{i+1}}{\to} \ldots, d^i d^{i-1} = 0 \ .$$

The categories of coherent sheaves are abelian ie are characterized by the existence of the exact sequences. Using McKay correspondence, we have the equivalence of the abelian category of coherent sheaves and the abelian category of quivers, whose vertices are complex Euclidean spaces, and the edges are mappings of these spaces Morphisms between quivers or phase transitions between D-branes are described by Ext groups

Substitution of orbifold charges $a = b = c = a' = b' = c' = 4$ in groups

$$Ext^0(Q, Q') = C^{aa'+bb'+cc'} \ ,$$





$$Ext^1(Q,Q^{'}) = C^{3ab+3bc+3ca} \ .$$

together with the Langlands hypothesis [4], gives the realization of the moduli space of superstring in terms of $SU(5)$ multiplets

$$3 \times (24 + 5_H + \overline{5}_H + 5_M + \overline{5}_M + 10_M + \overline{10}_M) \ .$$

Using this result makes it possible to construct a gauge-invariant superpotential of the MSSM model

$$W_{SU(5)} = \lambda^d_{ij} \cdot \overline{5}_H \times \overline{5}^{(i)}_M \times 10^{(j)}_M + \lambda^u_{ij} \cdot 5_H \times 10^{(i)}_M \times 10^{(j)}_M + \mu \cdot 5_H \times \overline{5}_H$$

The application of this superpotential together with five parameters determined from the experiment makes it possible to calculate the masses, the decay widths, and production cross sections of the superparticles of the supersymmetric theory - the theory of physics beyond the Standard Model connected with experiment at the Large Hadron Collider [5].

Rerefences

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