Conformal mappings in Hardy-type spaces

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Let $H^p(\mathbb{C}_+), 1 \leq p < +\infty$, [1] be the Hardy space of analytic in the half-plane $\mathbb{C}_+ = \{z : \Re z > 0\}$ functions, for which

$$\| f \| = \sup_{x>0} \left\{ \int\limits_{-\infty}^{+\infty} |f(x+iy)|^p dy \right\}^{1/p} < +\infty.$$

Let $D_\sigma = \{z : \Re z < 0, |\Im z| < \sigma\}$, $D^*_\sigma = \mathbb{C} \setminus \overline{D}_\sigma, \sigma > 0$.

Definition 1. Let $E^p(D_\sigma)$ and $E^p(D^*_\sigma)$, $1 \leq p < +\infty, \sigma > 0$, be the spaces of analytic functions in the domains D_σ and D^*_σ respectively, for which

$$\sup \left\{ \int\limits_{\gamma} |f(z)|^p |dz| \right\}^{1/p} < +\infty,$$

where supremum is taken over all segments γ, that are contained in D_σ and D^*_σ respectively.

We consider the properties of functions in the half-strip D_σ and in the exterior of half-strip D^*_σ. In [2] considered spaces $E^p(D_\sigma)$ and $E^p(D^*_\sigma)$ as spaces of signals. We propose a common point of viev on $E^p(D_\sigma)$ and $E^p(D^*_\sigma)$.

Theorem 2. *Function f belongs to $E^2(D^*_\sigma)$ if and only if, when the function*

$$F(w) = f\left(-w + \frac{2\sigma i}{\pi} \cos \frac{w\pi i}{2\sigma}\right) \sqrt{-1 + \sin \frac{w\pi i}{2\sigma}},$$

where $\sqrt{1} = 1$, belongs to $E^2(D_\sigma)$.

The proof of the theorem is based on the following lemma.

Lemma 3. *Function*

$$\tilde{w} = -w + \frac{2\sigma i}{\pi} \cos \frac{w\pi i}{2\sigma}$$

comformally maps D_σ into D^*_σ.

References

[1] Koosis, P.: Introduction to H^p spaces, Second edition. Cambridge Tracts in Mathematics. 115, Cambridge: Cambridge University Press, UK (1998).

[2] Dilnyi V., Huk K. Identification of Unknown Filter in a Half-Strip. Acta Applicandae Mathematicae. 2020. Vol.165. P.199-205.