The Terence Tao set and the Collatz conjecture

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The Collatz conjecture is an open problem in number theory stablished in 1937 by Lothar Collatz and can be stated as follows: If $f : \mathbb{N} \to \mathbb{N}$ is the function define by:

$$f(n) = \begin{cases} \frac{n}{2} & ; n \text{ is even} \\ 3n+1 & ; n \text{ is odd} \end{cases}$$

the conjecture says that given $n \in \mathbb{N}$, there exists $k > 0$ such that $f^{(k)}(n) = 1$ and the only orbit is $\{1, 2, 4\}$

In 2019, Terence Tao showed, in the context of the Collatz conjecture, that almost all $n \in \mathbb{N}$ belong to the set $W = \{n \in \mathbb{N} : min(O(n)) < f(n)\}$. In this paper we prove that the Collatz conjecture is true if and only if the set W is connected in $\mathbb{N}$ with the primal topology τ_f, where τ_f is the topology on $\mathbb{N}$ given by the open sets as those subset θ of $\mathbb{N}$ such that $f^{-1}(\theta) \subset \theta$.

References

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