On certain fractal-based estimations of subsidence volume

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In [1, 2], the particle size distribution $N_s(L > d_s)$ was defined as the number of particles being of any size L larger than d_s, where d_s runs over the real numbers. In the same way we can introduce the particle size distribution by volume $V_s(L > d_s)$ (and by mass $M_s(L > d_s)$) as the volume (mass) of particles being of any size L larger than d_s, where d_s runs over the real numbers. Certainly, $N_s(L > d_s)$, $V_s(L > d_s)$ and $M_s(L > d_s)$ are real functions. The particle size distribution $N_s(L > d_s)$ has fractal dimension D_s if

$$N_s(L > d_s) = \gamma {d_s}^{-D_s},$$

where γ is a constant coefficient.

Under some additional conditions of fractal nature of the loess soil and developing methods introduced in [3, 4, 5] we obtained certain predictive estimations of the coefficient of porosity after the disintegration of micro-aggregates. In this note we obtain some estimations of soil subsidence volume, based on the introduced above fractal dimension.

The particles forming the ground may have only a finite set of sizes. We denote these sizes $d_1, d_2, ..., d_{n-1}, d_n$ ranging in decreasing order from the largest. We assume that $\alpha = \alpha_j = d_j/d_{j-1}$, where $2 \leqslant j \leqslant n$, does not depend on j. This assumption corresponds to the idea of the self-similarity of fractal structures. In addition, all known mathematical fractals are constructed on this principle. As the structures formed by particles of a fixed size are self-similar, we also assume that all these structures have the same coefficient of porosity k_p as well as the same porosity $K_p = k_p/(1 + k_p)$. We discovered that under such conditions two different situations may occurred. Let k' be the coefficient of porosity and K' be the porosity of the soil after the disintegration of micro-aggregates.

Theorem 1. *In the above denotations we have :*

1. *if* $K_p \geq \alpha^{3-D_s}$ *then* $k' = \frac{(1+k_p)(\alpha^{3-D_s}-1)}{(\alpha^{3-D_s})^n-1} - 1$ *and* $K' = 1 - \frac{(\alpha^{3-D_s})^n-1}{(1+k_p)(\alpha^{3-D_s}-1)}$;
2. *if* $K_p < \alpha^{3-D_s}$ *then* $k' = \frac{k_p(1-\alpha^{3-D_s})}{1-(\alpha^{3-D_s})^n}$ *(5.18) and* $K' = \frac{k_p(1-\alpha^{3-D_s})}{k_p(1-\alpha^{3-D_s})+1-(\alpha^{3-D_s})^n}$.

The details of our experiments and techniques are described in [4].

References

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