Ultrametric spaces of $*$-measures

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Recall that an ultrametric on a set X is a metric d that satisfies the strong triangle inequality $d(x,y) \leq \max\{d(x,z), d(z,y)\}$, for all $x, y, z \in X$.

A triangular norm is a binary operation $*$ on the unit segment which is continuous, associative, commutative, monotone, for which 1 is the unit.

In [1], the functor M^* of $*$-measures acting on the category **Comp** of compact Hausdorff spaces is defined for any triangular norm $*$. A $*$-measures on a compact Hausdorff space X is a functional $\mu : C(X, [0,1]) \to [0,1]$ satisfying: 1) $\mu(c_X) = c, c \in [0,1]$, 2) $\mu(\varphi \vee \psi) = \mu(\varphi) \vee \mu(\psi)$, 3) $\mu(c * \varphi) = c * \mu(\varphi)$.

The set $M^*(X)$ of all $*$-measures on a compact Hausdorff space X is endowed with the weak* topology.

The space $M^*(X)$ of $*$-measures of compact support can be also considered for any Tychonov space X.

The aim of the talk is to consider the ultrametrization of the set $M^*(X)$ for any ultrametric space X. Given $r > 0$ we define the set $\mathcal{F}_r(X)$ of functions from $C(X, [0,1])$ constant on the balls of radius r.

Similarly as in [2] we define an ultrametric $\hat{d}$ on $M^*(X)$ by the formula $\hat{d}(\mu, \nu) = \inf\{r > 0 | \mu(\varphi) = \nu(\varphi)$ for all $\varphi \in \mathcal{F}_r(X)\}$.

We establish some topological and algebraic properties of the obtained ultrametric space $(M^*(X), \hat{d})$.

References

[1] Kh. Sukhorukova. Spaces of non-additive measures generated by triangular norms. *Proceedings of the International Geometry Center*, submitted.

[2] O.Hubal, M.Zarichnyi. Idempotent probability measures on ultrametric spaces. *Journal of Mathematical Analysis and Applications*, 343(2): 1052-1060, 2008.