The Golomb and Kirch topologies on the set of nonzero integers

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The *Golomb* (resp. *Kirch*) topology on $\mathbb{Z}$ is generated by the subbase consisting of arithmetic progressions $a + b\mathbb{Z}$ where $a \in \mathbb{Z}$ and b is a (square-free) number, coprime with a. It is known that the Golomb (Kirch) topology on the subspace $\mathbb{Z}^\bullet = \mathbb{Z} \setminus \{0\}$ of non-zero integers is Hausdorff and (locally) connected. In the talk we shall discuss the homeomorphisms of the Golomb and Kirch topologies on $\mathbb{Z}^\bullet$ and $\mathbb{N}$.

Theorem 1 (Banakh, Spirito, Turek)**.** *The space* $\mathbb{N}$ *with the Golomb topology has a unique self-homeomorphism.*

Theorem 2 (Banakh, Stelmakh, Turek)**.** *The space* $\mathbb{N}$ *with the Kirch topology has a unique self-homeomorphism.*

Theorem 3 (Spirito)**.** *The space* $\mathbb{Z}^\bullet$ *with the Golomb topology has exactly two self-homeomorphisms.*

Theorem 4 (Stelmakh)**.** *The space* $\mathbb{Z}^\bullet$ *with the Kirch topology has exactly two self-homeomorphisms.*

References

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