The relation between T_0-topologies with the weight $2^{n-2} < k \le 2^{n-1}$ on n-element set and T_0-topologies close to the discrete on $(n-1)$-element set

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It is common to speak that the topology on n-element set X has the weight k (or it belongs to k-class), if the topology contains k elements. Let us designate the minimum neighborhood of the element $a \in X$ by M_a. The concept of the vector of the topology (the nondecreasing sequence of the reduced by 1 powers of the minimum neighborhoods of all elements of X) have been introduced in [3]. In the work [3] the theorem on three types of the vectors of T_0-topologies with the weight $2^{n-1} < k \le 2^n$ (close to the discrete topologies) has been proved:

1. $(0, ..., 0, \alpha_n)$, $1 \le \alpha_n \le n-1$;
2. $(\underbrace{0, ..., 0}_{k}, 1, ..., 1)$, $1 \le k \le n-2$, and $\bigcap_{m=k+1}^{n} M_m = \{y\}$;
3. $(0, ..., 0, 1, 1)$, $M_{n-1} \bigcap M_n = \varnothing$.

If T_0-topology on n-element set induces close to the discrete T_0-topology on some $(n-1)$-element set, then such topologies are called consistent.

The fact that T_0-topologies with the vectors $(0, ..., 0, \alpha_{n-1}, \alpha_n)$, $1 \le \alpha_{n-1} \le n-2$, $2 \le \alpha_n \le n-1$ (consistent with the close to discrete topologies of the first type) have weight $2^{n-2} < k \le 2^{n-1}$ has been shown in [4]. The obtained results connected with the enumeration of T_0-topologies and the calculation of T_0-topologies in the individual classes have been compared with the results [1], [2].

T_0-topologies with the weight $2^{n-2} < k \le 2^{n-1}$, which are consistent with the close to discrete topologies of the second and the third types have been considered in this paper. The fallowing facts have been proved: these topologies do not form new classes, and such topologies are contained in the same classes as T_0-topologies with vectors $(0, ..., 0, \alpha_{n-1}, \alpha_n)$.

References

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