The surfaces with the flat normal connection and the constant curvature of Grassmann image in Minkowski space

Polina Stegantseva
(ZNU, Zaporizhzhya, Ukraine)
E-mail: `stegpol@gmail.com`

Marina Grechneva
(ZNU, Zaporizhzhya, Ukraine)
E-mail: `grechnevamarina@gmail.com`

The use of the concept of Grassmann image of the surface extends the circle of the problems, and it is the one of the methods of the study of the differential geometry of the surface. One the problems is the problem connected with the proof of the existence of the surface with given properties of its Grassmann image. In this paper, we consider the existence of the surface with the flat normal connection and constant curvature of its Grassmann image in Minkowski space ${}^1R_4$. The results of the solution of this problem depend on the type of Grassmann image. The concept of the normal connection of the submanifold of Riemannian manifold has been introduced by E. Cartan. The submanifolds with the flat normal connection have zero torsion. The important property of the surfaces with the flat normal connection is the existence of the parameterization, in which the first and two second quadratic forms can be reduced to the diagonal form simultaneously. The surfaces with the flat normal connection and their images in Minkowski space also have the additional properties.

If a time-like surface $V^2 \subset^1 R_4$ has the flat normal connection and the non-degenerate Grassman image, then that Grassman image is the time-like surface. In case of the space-like surface with a flat normal connection, its Grassmann image can be either a space-like surface or a time-like one.

The following existence theorems have been proved in this paper.

Theorem 1. *Let any $k \in [0,1]$ is given. Then in the space ${}^1R_4$ there exists the time-like C^3 class surface with the flat normal connection and the non-degenerate Grassmann image with the constant curvature $\bar{K} = k$. In the case $k = 0$, there is a surface with a constant Gauss curvature $K = 0$; if $k \in (0,1]$, then there exists the surface with the given function of the Gauss curvature $K = (\alpha_0^2 + 1)\beta(u^1)\delta(u^2)$, where $\alpha_0 = const, \beta(u^1), \delta(u^2))$ - the continuous functions.*

Theorem 2. *Let any $k \in (-\infty, -1]$ ($k \in (0, +\infty)$) is given. Then in the space ${}^1R_4$ there exists the space-like C^3 class surface with the flat normal connection and the non-degenerate space-like (time-like) Grassman image with constant curvature $\bar{K} = k$. If $k = 0$, then there exists the surface with the constant Gauss curvature $K = 0$; in the other cases there exists the surface with the given function of Gauss curvature $K(u^1, u^2) = (1 - \alpha_0^2)\beta(u^1)\delta(u^2)$, where $\alpha_0 = const, \alpha_0 \neq \pm 1$, $\beta(u^1), \delta(u^2)$ - the continuous functions.*

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