Some generalizations of the known theorems of the type of geodesical unique definability

Helena Sinyukova
(State institution «South Ukrainian National Pedagogical University named after K. D. Ushinsky»)
E-mail: `olachepok@ukr.net`

The realized in [1] broadening to the noncompact but complete spaces of affine connection the well-known Hopf-Bochner-Uano techniques ([3], for example) on the grounding the so called vanishing theorems allowed to broad to the corresponding spaces some well-known theorems of the type of geodesical unique definability ([2], for example). In particular, it is grounded that the next theorems take place.

Theorem 1. *Complete connected noncompact Riemannian C^r-space V^n $(n > 2,\ r > 4)$ with the positive defined metric tensor and the Einstein tensor that doesn't equal to zero identically, that satisfies the recurrence conditions*

$$T^{(\alpha\beta)}_{ijkl,mh} g^{mj} g^{hl} E^{ik}_{..} = T^{(\alpha\beta)}_{ijkl} W^{ijkl} + \frac{1}{n} T^{(\gamma j)}_{ijkl} R^{(\alpha|l|\beta)}_{\gamma\ \cdot\ \cdot} E^{ik}_{..} - \frac{1}{n} T^{(\alpha j)}_{ijkl} R^{\beta l}_{..} E^{ik}_{..} -$$

$$- \frac{1}{n} T^{(\beta j)}_{ijkl} R^{\alpha l}_{..} E^{ik}_{..} + T^{(\alpha\beta)}_{ijkl,m} W^{ijklm},$$

where

$$T^{\alpha\beta}_{ijkl} = n \left(\delta^{\beta}_{j} R^{\alpha}_{ikl} - \delta^{\beta}_{k} R^{\alpha}_{lji} \right) - g_{ik} \left(\delta^{\beta}_{j} R^{\alpha}_{.l} - R^{\alpha\ \ \beta}_{jl\ \cdot} \right) + g_{jl} \left(\delta^{\beta}_{k} R^{\alpha}_{.i} - R^{\alpha\ \ \beta}_{ki\ \cdot} \right),$$

"," means the corresponding covariant differentiation, doesn't admit non-trivial (different from the affine) geodesic mappings in the large.

Theorem 2. *Complete connected noncompact Riemannian C^r-space V^n $(n > 2,\ r > 4)$ with the positive defined metric tensor and the Einstein tensor that doesn't equal to zero identically, that satisfies the recurrence conditions*

$$P^{(\alpha\beta)}_{ij,kh} g^{hi} E^{kj}_{..} = P^{(\alpha\beta)}_{ij,k} W^{ijk} + P^{(\alpha\beta)}_{ij} W^{ij},$$

where

$$P^{\alpha\beta}_{ij} = \delta^{\beta}_{i} R^{\alpha}_{.j} - \delta^{\beta}_{j} R^{\alpha}_{.i},$$

W^{ij} and W^{ijk} are some arbitrary tensors, correspondingly of the second and the third valence, doesn't admit non-trivial (different from the affine) geodesic mappings in the large.

Examples of the corresponding spaces are given.

References

[1] Pigola S., Rigoli M., Mikes J. *Vanishing in finitness results in geometric analysis.* in *A Generalization of the Bochner Technique.*, Berlin: Birkhauser Varlag AG, 2008

[2] Sinyukova, H.N. Geodesic uniqueness in the whole of some generally recurrent Riemannian spaces, *Journal of Mathematical Sciences*, 177(5) : 710–715, 2011.

[3] Wu H.H. *The Bochner technique in differential geometry.* Harwood: Harwood Acad. Publ.,1987