About one class of continual approximate solutions with arbitrary density

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The kinetic Boltzmann equation is one of the central equations in classical mechanics of many-particle systems. For the model of hard spheres it has a form [1, 2]:

$$D(f) = Q(f, f). \tag{1}$$

We will consider the continual distribution [3]:

$$f = \int\limits_{\mathbb{R}^3} du \int\limits_0^{+\infty} d\rho \, \varphi(t, x, u, \rho) M(v, u, x, \rho), \tag{2}$$

which contains the local Maxwellian of special form describing the screw-shaped stationary equilibrium states of a gas (in short-screws or spirals). They have the form:

$$M(v, u, x, \rho) = \rho \left(\frac{\beta}{\pi} \right)^{\frac{3}{2}} e^{-\beta(v-u-[\omega\times x])^2}. \tag{3}$$

Physically, distribution (3) corresponds to the situation when the gas has an inverse temperature $\beta = \frac{1}{2T}$ and rotates in whole as a solid body with the angular velocity $\omega \in R^3$ around its axis on which the point $x_0 \in R^3$ lies,

$$x_0 = \frac{[\omega \times u]}{\omega^2}, \tag{4}$$

The square of this distance from the axis of rotation is

$$r^2 = \frac{1}{\omega^2} [\omega \times (x - x_0)]^2, \tag{5}$$

ρ is the arbitrary density, $u \in R^3$ is the arbitrary parameter (linear mass velocity for x), for which $x || \omega$, and $u + [\omega \times x]$ is the mass velocity in the arbitrary point x. The distribution (3) gives not only a rotation, but also a translational movement along the axis with the linear velocity

$$\frac{(\omega, u)}{\omega^2} \omega,$$

Thus, it really describes a spiral movement of the gas in general, moreover, this distribution is stationary (independent of t), but inhomogeneous.

The purpose is to find such a form of the function $\varphi(t, x, u, \rho)$ and such a behavior of all hydrodynamical parameters so that the uniform-integral remainder [3]

$$\Delta = \sup_{(t,x)\in\mathbb{R}^4} \int\limits_{\mathbb{R}^3} |D(f) - Q(f, f)| dv, \tag{6}$$

and its modification "with a weight":

$$\widetilde{\Delta} = \sup_{(t,x)\in\mathbb{R}^4} \frac{1}{1 + |t|} \int\limits_{\mathbb{R}^3} |D(f) - Q(f, f)| dv, \tag{7}$$

become vanishingly small.

Also some sufficient conditions to minimization of remainder Δ and $\widetilde{\Delta}$ are found. In this work we succeeded a few to generalize results, which obtained in [3]. The obtained results are new and may be used with the study of evolution of screw and whirlwind streams.

References

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[3] V.D. Gordevskyy, E.S. Sazonova. Continual approximate solution of the Boltzmann equation with arbitrary density. *Matematychni Studii.*, 45(2) : 194–204, 2016.