Fuzzy ultrametrization of spaces of non-additive measures on fuzzy ultrametric spaces

Aleksandr Savchenko
(Kherson State University, Kherson, Ukraine)
E-mail: savchenko.o.g@ukr.net

Recall that a triangular norm is a binary operation $*$ on the unit segment which is continuous, associative, commutative, monotone, for which 1 is the unit. The following are examples of triangular norms: min, $\cdot$ (multiplication), $a * b = \max\{a + b - 1, 0\}$ (Łukasiewicz t-norm).

Given a triangular norm $*$, we define a fuzzy metric on a set X as a function $M\colon X \times X \times (0, \infty) \to (0, 1]$ satisfying for all $x, y, z \in X$ and $s, t \in (0, \infty)$:

(1) $M(x, y, t) > 0$;
(2) $M(x, y, t) = 1$ if and only if $x = y$;
(3) $M(x, y, t) = M(y, x, t)$;
(4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;
(5) the function $M(x, y, \cdot)\colon (0, \infty) \to [0, 1]$ is continuous

(see, e.g., [2]).

A fuzzy metric is said to be a fuzzy ultrametric (a fuzzy non-Archimedean metric) if $* = \min$ and the following holds: (4') $M(x, y, t) * M(y, z, s) \leq M(x, z, \max\{t, s\})$. This is known to be equivalent to the following: (4") $M(x, y, t) * M(y, z, t) \leq M(x, z, t)$.

By $I(X)$ we denote the set of all idempotent measures on a compact Hausdorff space X (see [5]). This set is endowed with the weak* topology. In this way we obtain a functor in the category of compact Hausdorff spaces and continuous maps.

A standard construction allows us to consider the set of idempotent measures of compact support for any Tychonov space X; we keep the notation $I(X)$ for this set.

Let (X, M) be a fuzzy ultrametric space. A fuzzy ultrametric $\bar{M}$ on the set $I(X)$ is defined in [3]. The construction $(I(X), \bar{M})$ determines a functor in the category of fuzzy ultrametric spaces and non-expanding maps.

We continue the investigations of the mentioned paper as follows. The idempotent measure monad on the category of fuzzy ultrametric spaces is an idempotent counterpart of the probability measure monad on the same category which is introduced and investigated in [4]. Also, one can prove analogous results for the functor and monad of another class of non-additive measures, namely the max-min measures (see, e.g., [1]).

References

[1] V. Brydun, A. Savchenko, M. Zarichnyi. Fuzzy metrization of the spaces of idempotent measures. *European Journal of Mathematics.* 6(1) : 98–109, 2020.
[2] A. George, P. Veeramani. On some results in fuzzy metric spaces. *Fuzzy Sets Syst.* 64 : 395–399, 1994.
[3] Changqing Li, Z. Yang, Fuzzy Ultrametrics Based on Idempotent Probability Measures, *The Journal of Fuzzy Mathematics.* 22(2) : 463–476, 2014.
[4] A. Savchenko, M. Zarichnyi. Probability Measure Monad on the Category of Fuzzy Ultrametric Spaces. *Azerbaijan Journal of Mathematics*, 1(1) : 114–121, 2011.
[5] M. M. Zarichnyi. Spaces and maps of idempotent measures. *Izv. RAN. Ser. Mat.* 74(3) : 45–64, 2010