On countable multiplicity of mappings

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A number of papers of mathematicians are devoted to the study of countable-to-one mappings, in particular M.M. Luzin, P.S. Alexandrov, A.M. Kolmogorov, B.O. Pasynkov, Yu.Yu. Trokhymchuk. In [1] a dense open set of points of local homeomorphism exists for any countable-to-one continuous mapping of two manifolds of equal dimensions was proved. Moreover, for the existence of a dense set of points of local homeomorphism, it suffices to require countable multiplicity of zero-dimensional mapping, even for points of some subset of the second category in the image [2]. In the one-dimensional case, the statement of the theorem remains valid for nowhere constant functions of the first Baire class with the Darboux property and with the set of countable levels of the second category in the image [3]. In paper [4] we consider the class of continuous on $[0,1]$ functions preserving digit 1 in three-symbol Q_3–representation of a number and prove that any such function is countable-to-one and it has at most two infinite level sets. If we neglect some set of the first category, then with countable-to-one arbitrary B-measurable mapping of complete separable zero-dimensional uncountable space there exists a dense set of points of local homeomorphism [5]. It turns out that if quasi-continuous mapping of two complete separable metric spaces with the set of countable levels of the second category is nearly continuous on dense open set and is semi-open and pre-open, then it has a dense open set of points of local homeomorphism.

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