THE ANALOGUE OF DARBOUX EQUATION IN GALILEAN SPACE

Safarov Tulqin Nazarovich
(Termez State University (Uzbekiston))
E-mail: `tolqin.1986@mail.ru`

Let the surface F of the class $C^k(k \geq 2)$ in R_3^1 be given by the vector function $r = r(u, v)$ in the region $D \in R_2^1$, we will assume that the Cartesian coordinates R_3^1 are entered in x, y, z, and let $k$be the unit vector of the axis z. Each of the coordinates of the vector $r(u, v) = \{u,\ y(u, v),\ z(u, v)\}$ satisfies a certain differential equation. Let's deduce it, for example, for function $z(u, v)$.

Obviously, $z(u, v) = (r(u, v),\ k)$. Then

$$z_u = (r_u, k)\,,\ \ z_v = (z_v, k)\,,\ \ z_{uu} = (r_{uu}, k),\ \ z_{uv} = (r_{uv}, k),\ \ z_{vv} = (r_{vv}, k).$$

Using derivation formulas in R_3^1 we get

$$z_{uu} = \Gamma_{11}^2 z_v + L(n, k), \quad z_{uv} = \Gamma_{12}^2 z_v + M(n, k), \quad z_{vv} = \Gamma_{22}^2 z_v + N(n, k) \tag{1}$$

Where L, M, N is the coefficients of the second quadratic form, n is the surface normal.

If we introduce the notation

$$z_{11} = z_{uu} - \Gamma_{11}^2 z_v, \quad z_{12} = z_{uv} - \Gamma_{12}^2 z_v, \quad z_{22} = z_{vv} - \Gamma_{22}^2 z_v \tag{2}$$

then from (**??**) and (**??**) we obtain

$$z_{11} = L(n, k), \quad z_{12} = M(n, k), \quad z_{22} = N(n, k) \tag{3}$$

The unit normal vector is determined by the formula

$$n = \left\{0,\ \frac{z_v}{\sqrt{y_v^2 + z_v^2}},\ -\frac{y_v}{\sqrt{y_v^2 + z_v^2}}\right\}.$$

We have

$$(n, k) = -\frac{y_v}{\sqrt{y_v^2 + z_v^2}} \tag{4}$$

the for formula above gives a determination for the unit normal vector.

From (**??**) and (**??**) we obtain.

$$L = -\frac{z_{11}}{y_v}\sqrt{y_v^2 + z_v^2}, \quad M = -\frac{z_{12}}{y_v}\sqrt{y_v^2 + z_v^2}, \quad N = -\frac{z_{22}}{y_v}\sqrt{y_v^2 + z_v^2} \tag{5}$$

From equalities (**??**) and the formula for the Gaussian curvature $K = \frac{LN - M^2}{G}$ we obtain the Darboux equation $z_{11} z_{22} - z_{12}^2 = y_v^2 K$.

References

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[2] I.Ya.Bakelman, A.L. Verner, B.E. Kantor. An introduction to differential geometry "in the large". – Moscow: "Nauka", 1973.