On the τ-placedness of space of the permutation degree

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We shall say that a set P is of the type G_τ in X if there exists a family $\gamma = \{U_\alpha : \alpha \in A, \mid A \mid \leq \tau\}$ of open sets in X such that $\bigcap_{\alpha \in A} U_\alpha = P$ (taken from [1]).

A subset $A \subset X$ is said to be τ-placed in X, if for each $x \in X \setminus A$ there exists a set $P \subset X$ of type G_τ in X such that $x \in P \subset X \setminus A$ (taken from [1]).

A permutation group X is the group of all permutations (i.s. one-one and onto mappings $X \to X$). A permutation group of a set X is usuallay denoted by $S(X)$. If $X = \{1, 2, ..., n\}$, then $S(X)$ is denoted by S_n, as well.

Let X^n be the n-th power of a compact X. The permutation group S_n of all permutations, acts on the n-th power X^n as permutation of coordinates. The set of all orbits of this action with quotient topology we denote by SP^nX. Thus, points of the space SP^nX are finite subsets (equivalence classes) of the product X^n. Thus two points $(x_1, x_2, ..., x_n), (y_1, y_2, ..., y_n) \in X^n$ are considered to be equivalent if there is a permutation $\sigma \in S_n$ such that $y_i = x_(\sigma(i))$ for all $i = 1, 2, ..., n$. The space SP^nX is called n-permutation degree of a space X. Equivalent relation by which we obtained space SP^nX is called the symmetric equivalence relation. The n-th permutation degree is always a quotient of X^n. Thus, the quotient map is denoted by as following: $\pi_n^s : X^n \to SP^nX$. Where for every $x = (x_1, x_2, ..., x_n) \in X^n$, $\pi_n^s((x_1, x_2, ..., x_n)) = [(x_1, x_2, ..., x_n)]$ is an orbit of the point $x = (x_1, x_2, ..., x_n) \in X^n$.

The concept of a permutation degree has generalizations. Let G be any subgroup of the group S_n. Then it also acts on X^n as group of permutations of coordinates. Consequently, it generates a G-symmetric equivalence relation on X^n. This quotient space of the product of X^n under the G-symmetric equivalence relation is called G-permutation degree of the space X and it is denoted by SP_G^nX. An operation SP_G^n is also the covariant functor in the category of compacts and it is said to be a functor of G-permutation degree. If $G = S_n$, then $SP_G^n = SP^n$. If the group G consists only of unique element, then $SP_G^nX = X^n$ (taken from [2]).

Theorem 1. *If the set SP^nA is τ-placed in SP^nX, then the set $(\pi_n^s)^{-1}(SP^nA)$ is also τ-placed in X^n.*

References

[1] A.V.Arkhangel'skii, Topological function spaces. Math. its Appl., vol. 78, Dordrecht: Kluwer, 1992, 205 p. ISBN: 0-7923-1531-6 . Original Russian text published in Arkhangel'skii A.V. Topologicheskie prostranstva funktsii, Moscow: MGU Publ., 1989, 222 p.

[2] V.V.Fedorchuk, V.V.Filippov, Topology of hyperspaces and its applications. // Mathematica, cybernetica. Moscow: 4 (1989) - 48 p.