On a result of G. Pisier concerning Sidon sets

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Let G be a compact abelian group, Γ be its dual group, i.e., the set of all continuous characters on G. $C(G)$, $L_p(G), 0 < p \leq \infty$, are classical Banach spaces (integrals are considered with respect to the Haar measure on G). $M(G)$ denotes the Banach space of all regular Borel measures on G. Convolution operations are defined by the usual ways and are denoted by $\varphi \star f$, $\mu \star f$ for functions f, φ and a measure $\mu \in M(G)$. Below, H denotes a Hilbert space, $S_p(H)$, for $p \in (0, \infty)$, is the space of operators in H from Shatten-von Neumann class S_p (operators, whose singular numbers are in the classical space l_p). By S_∞ we denote the space of *all* operators in H. X, Y are Banach spaces, $L(X, Y)$ is a Banach space of all bounded linear operators from X to Y.

Definition 1. An operator $T : X \to Y$ *can be factored through an operator* from $S_p(H)$ (*through an* S_p*-operator*), if there are operators $A \in L(X, H)$, $U \in S_p(H)$ and $B \in L(H, Y)$ such that $T = BUA$. If T can be factored through an operator from $S_p(H)$, then we put $\gamma_{S_p}(T) = \inf ||A||\, \sigma_p(U)\, ||B||$, where the infimum is taken over all possible factorizations of T through an operator from $S_p(H)$.

In [1], Giles Pisier gave a geometric characterization of Sidon subsets of Γ (for the definitions and formulations see [1, §4.b]). One of the main tool in his proof was the following result: For a function $f \in C(G)$ and a convolution operator $\star f : M(G) \to C(G)$, the necessary and sufficient condition for the set of Fourier coefficients $\hat{f} := \{\hat{f}(\gamma)\}$ to be absolutely summable is that the operator $\star f$ can be factored through a Hilbert space. It is clear that the last condition is the same as the condition "the operator $\star f$ can be factored through an S_∞-operator".

We present some generalizations of this result (proving simultaneously the above one). In particular, we have

Theorem 2. *Let* $f \in C(G)$, $0 < q \leq 1$ *and* $1/p = 1/q - 1$. *Consider a convolution operator* $\star f : M(G) \to C(G)$. *The set* $\hat{f}$ *of Fourier coefficients of* f *belongs to* l_q *if and only if the operator* $\star f$ *can be factored through a Schatten-von Neumann* S_p*-operator in a Hilbert space. Moreover, if* $\hat{f} \in l_q$, *then* $\gamma_{S_p}(\star f) = (\sum_{\gamma \in \Gamma} |\hat{f}(\gamma)|^q)^{1/q}$. *On the other hand,* $|| \star f|| = ||f||_{C(G)}$.

Instead of $M(G)$, we can consider the spaces $L_p(G)$ in the theorem (changing some values of parameters). Also, we can get some similar results for the factorizations of the convolution operators through the operators of the Lorentz-Schatten classes $S_{r,p}$ (associated with the Lorentz sequences spaces $l_{r,p}$).

References

[1] Giles Pisier. *Factorization of Linear Operators and Geometry of Banach Spaces*, volume 60 of *CBMS*. Amer. Math. Soc., Providence, Rhode Island, 1985.