Uniform measures in Euclidean space

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A uniform measure in Euclidean space $\mathbb{R}^d$ is a measure that assigns to each ball $B(x,r)$ with center x in the support of the measure, a mass dependent of r and independent of the choice of x.

For example any invariant measure of a subgroup of the isometry group of $\mathbb{R}^d$ is uniform, and this sub-class of uniform measures are called homogeneous measures. There are known a few examples of non-homogeneous uniform measures, such as the volume measure of the "light cone" $\{x^2+y^2+z^2 = w^2\} \subset \mathbb{R}^4$.

The study of uniform measures in Euclidean space was initiated by David Preiss as the crucial ingredient of his 1987 proof of the Besicovitch conjecture [4], and one motivation for extending this study is to understand the structure of measures in general geometry. It is known (see [1]) that a uniform measure must be a multiple of the k-dimensional area measure restricted to a k-dimensional analytic variety, and the classification of k-dimensional uniform measures remains a difficult open problem, still open even in the plane (see also [2], [3]). I will present a classification [5] of 1-dimensional uniform measures in $\mathbb{R}^d$, and mention some open questions for more general dimensions. This is joint work with Paul Laurain, from Paris 7 University.

References

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