The local τ-density of a linearly ordered spaces

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A set $A \subset X$ is said to be dense (in X), if $[A] = X$. The density of the space X is defined as the smallest cardinal $|A|$, where A is a dense subset of X [1]. This cardinal is denoted by $d(X)$. If $d(X) = \tau$, $\tau \geq \aleph_0$, the space X is said to be $\tau-$dense. If $d(X) \leq \aleph_0$, then X is said to be separable.

A topological space X is called locally $\tau-$dense at the point $x \in X$, if τ is the smallest cardinal number, such that x has a neighbourhood of density τ in X [2]. Local density at x is denoted by $ld(x)$. Local density of the space X is defined as follows:

$$ld(X) = \sup\{ld(x) : \ x \in X\}.$$

It is clear that local density of a topological space cannot exceed the density of said space, i.e. $ld(X) \leq d(X)$.

We say that the weak density of the topological space is $\tau \geq \aleph_0$, if τ is the smallest cardinal number such that there exists a π-base coinciding with τ of centered systems of open sets, i.e. there is a π-base $B = \cup\{B_\alpha : \alpha \in A\}$ where B_α is a centered system of open sets for each $\alpha \in A, |A| = \tau$ [3]. Weak density of topological space X is denoted by $wd(X)$.

Topological space X is said local weak τ-dense at a point x, if τ is the smallest cardinal number such that x has a neighborhood of weak density τ in X [4]. Local weak density at a point x is denoted by $lwd(x)$. The local weak density of a topological space X is defined as the supremum of all numbers $lwd(x)$ for $x \in X$:

$$lwd(X) = sup\{lwd(x) : x \in X\}.$$

It is clear that local weak density of a topological space cannot exceed the weak density of said space, i.e. $lwd(X) \leq wd(X)$.

Let X be a set, and $<$ be some relation on X. We say that $<$ is a linear order on X if the relation $<$ satisfies the following properties:

1) If $x < y$ and $y < z$, then $x < z$;
2) If $x < y$ then the relation $y < x$ does not hold;
3) If $x \neq y$ then either $x < y$ or $y < x$ holds.

A set X together with some linear order defined on it is called a linearly ordered set [1].

Theorem 1. *Suppose that a space X satisfies at least one of the following conditions:*

1) X is a linearly ordered topological space with the interval topology,

2) X is pseudometric space.

Then X is locally $\tau-$dense if and only if it is locally weak $\tau-$dense.

References

[1] R. Engelking. General Topology. *Heldermann Verlag*, Berlin, 1989. Zbl 0684.54001.

[2] T.K. Yuldashev, F.G. Mukhamadiev. *Local τ-density and local weak τ-density in topological spaces*, volume 18(1) of *Siberian Electronic Mathematical Reports*. pp. 474-478. 2021.

[3] R.B. Beshimov. *Some cardinal properties of topological spaces connected with weakly density*, volume 10:3 of *Methods Funct. Anal. Topology*. pp. 17-22. 2004.

[4] T.K. Yuldashev, F.G. Mukhamadiev. *The local density and the local weak density in the space of permutation degree and in Hattori space*, volume 6(2) of *Ural Mathematical Journal*. pp. 108-116. 2020.