Twistor spaces on foliated manifolds

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Let M^{2n} be an even-dimensional Riemannian manifold, the twistor space $Z(M)$ is the parametrizing space for compatible almost complex structures on M. It is a bundle over M, with fiber $SO(2n)/U(n)$ and is equipped with two almost complex structures $J^{\pm}$, where J^{+} can be integrable but J^{-} is never integrable, however, it still is important as will be discussed. Moreover, in the case where J^{+} is integrable, it is shown in [1] that M has particular properties, especially when $n = 2$, which is an interesting case in physics, since the holomorphic structure of the twistor space correspond to a conformal structure of M. This correspondence is called the Penrose correspondence.

This talk is based on a joint work with R. A. Wolak [2] , in which, the theory of twistors on foliated manifolds is developed. We construct the twistor space of the normal bundle of a foliation. It is demonstrated that the classical constructions of the twistor theory lead to foliated objects and permit to formulate and prove foliated versions of some well-known results on holomorphic mappings. Since any orbifold can be understood as the leaf space of a suitably defined Riemannian foliation we obtain orbifold versions of the classical results as a simple consequence of the results on foliated mappings.

References

[1] Michael F. Atiyah, Nigel J. Hitchin and Isadore M. Singer, Self-duality in four-dimensional Riemannian geometry, *Proc. R. Soc. Lond. A* **362** (1978) 425–461.

[2] Rouzbeh Mohseni, Robert A. Wolak. Twistor spaces on foliated manifolds. arXiv:2009.10491