Hyperspaces of convex sets related to idempotent mathematics

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The notion of hyperspace is one of the most important not only in topology but also in another parts of mathematics. This notion allows us to consider multivalued maps as single valued with the values being points of a hyperspace.

The hyperspace of compact convex sets in compact convex subsets of Euclidean spaces was first considered by Nadler, Quinn, and Stavrokas [4]. They proved, in particular, that the hyperspace of the Euclidean space $\mathbb{R}^n$, $n \geq 2$, is homeomorphic to the punctured Hilbert cube.

We denote by $x \oplus y$ the coordinatewise maximum of $x, y \in \mathbb{R}^n$. Given $t \in \mathbb{R}$ and $(y_1, \dots, y_n) \in \mathbb{R}^n$, let $t \otimes (y_1, \dots, y_n) = (\min\{t, y_1\}, \dots, \min\{t, y_n\})$. A subset $A \subset \mathbb{R}^n$ is said to be max-min convex if, for any $x, y \in A$ and any $t \in \mathbb{R}$, we have $x \oplus t \otimes y \in A$. It is proved in [3] that the hyperspace of compact max-min convex sets in the Euclidean space $\mathbb{R}^n$, $n \geq 2$, is homeomorphic to the punctured Hilbert cube.

Following the style of idempotent mathematics we define, for any $t \in \mathbb{R}$ and any $x = (x_1, \dots, x_n) \in \mathbb{R}^n$, $t \odot x = (t + x_1, \dots, t + x_n)$. A subset $A \subset \mathbb{R}^n$ is said to be max-plus convex (see, e.g., [1]) if, for any $x, y \in A$ and any $t \in (-\infty, 0]$, we have $x \oplus t \odot y \in A$. It is proved in [3] that the hyperspace of compact max-min convex sets in the Euclidean space $\mathbb{R}^n$, $n \geq 2$, is homeomorphic to the punctured Hilbert cube.

Recall that the Fell topology on the hyperspace of closed subsets of a Hausdorff topological space has as a subbase all sets of the form $\{A \mid A \cap V \neq \emptyset\}$, where V is an open subset of X, and also all sets of the form $\{A \mid A \subset W\}$, where W has compact complement. We denote by $\mathrm{Mpcc}_F\mathbb{R}^n$ and $\mathrm{Mmcc}_F\mathbb{R}^n$ the hyperspaces of the max-plus convex and max-min convex nonempty closed (not necessarily bounded) subsets of $\mathbb{R}^n$ endowed with Fell topology. See [5] for description of topology of the hyperspaces of compact convex subsets of $\mathbb{R}^n$ endowed with Fell topology.

Every non-empty closed subset A of a metric space (X, d) can be identified with the distance function $x \mapsto d(x, A)$. The topology of convergence on bounded sets induces the Attouch-Wetts topology on the hyperspace of non-empty closed sets. We denote by $\mathrm{Mpcc}_{AW}\mathbb{R}^n$ and $\mathrm{Mmcc}_{AW}\mathbb{R}^n$ the hyperspaces of the max-plus convex and max-min convex nonempty closed (not necessarily bounded) subsets of $\mathbb{R}^n$ endowed with Attouch-Wetts topology. Some results on ANR-properties of the hyperspaces in the Attouch-Wetts topology can be found in [6].

The aim of the talk is to discuss properties of the hyperspaces $\mathrm{Mpcc}_F\mathbb{R}^n$ and $\mathrm{Mmcc}_F\mathbb{R}^n$, $\mathrm{Mpcc}_{AW}\mathbb{R}^n$, and $\mathrm{Mmcc}_{AW}\mathbb{R}^n$.

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