Second classical Zariski topology of multiplicational module

Marta Maloid-Hliebova
(Ivan Franko National University of Lviv, 1, Universytetska St., Lviv, 79000, Ukraine)
E-mail: martamaloid@gmail.com

Let R be a associative ring and M an multiplicative R-module. If N is a subset of an R-module M we write $N \leq M$ to indicate that N is a submodule of M.

Definition 1. Proper submodule P of the left module M is called **prime submodule**, if quotient module M/P is prime left module, ie $Ann(K/P) = Ann(M/P)$ for every nonzero submodule K/P of module M/P.

This definition can be found in such papers: [1], [2], and there are a lot of interesting results about such modules. Set of all prime submodules of module M is called prime spectrum of module M and is denoted by $Spec(M)$.

Definition 2. A non-zero submodule N of M is said to be second if for each $a \in R$, the homomorphism $N \to^a N$ is either surjective or zero [3]. More information about this class of modules can be found in [4].

Let $Spec^s(M)$ be the set of all second submodules of M. For any submodule N of M, $V^{s*}(N)$ is defined to be the set of all second submodules of M contained in N. Of course, $V^{s*}(0)$ is just the empty set and $V^{s*}(M)$ is $Spec^s(M)$. It is easy to see that for any family of submodules $N_i (i \in I)$ of M, $\cap_{i \in I} V^{s*}(N_i) = V^{s*}(\cap_{i \in I} N_i)$. Thus if $\zeta s*(M)$ denotes the collection of all subsets $V^{s*}(N)$ of $Spec^s(M)$, then $\zeta s*(M)$ contains the empty set and $Spec^s(M)$, and $\zeta s*(M)$ is closed under arbitrary intersections. In general $\zeta s*(M)$ is not closed under finite unions.
Now let N be a submodule of M. We define $W^s(N) = Spec^s(M) - V^{s*}(N)$ and put $\Omega^s(M) = \{W^s(N) : N \leq M\}$. Let $\eta^s(M)$ be the topology on $Spec^s(M)$ by the sub-basis $\Omega^s(M)$. In fact $\eta^s(M)$ is the collection U of all unions of finite intersections of elements of $\Omega^s(M)$ [6]. We call this topology the second classical Zariski topology of M.

Theorem 3. *Let R be a associative Noetherian ring and let M be a cotop multiplicational R-module with finite length. Assume that the second classical Zariski topology of M and the Zariski topology of M considered in* [5] *coincide. Then M is a comultiplication R-module.*

Theorem 4. *Let R be a associative Noetherian ring and let M be a co-multiplication R-module with finite length. Then $Spec^s(M)$ is a spectral space (with the second classical Zariski topology).*

References

[1] Page S. *Properties of quotient rings* // Can. J. Math. – 1972 – 24, №6 – P. 1122-1128.
[2] Dauns J. *Prime modules* // Reine Angew. Math. – 1978. – 298. – P. 156-181.
[3] S. Yassemi, *The dual notion of prime submodules*, // Arch. Math (Brno) – 2001. – 37. – P. 273–278.
[4] H. Ansari-Toroghy and F. Farshadifar, *On the dual notion of prime submodules*, to appear in Algebra Colloq.
[5] H. Ansari-Toroghy and F. Farshadifar, *he Zariski topology on the second spectrum of a module*, to appear in Algebra Colloq.
[6] J. R. Munkres *Topology. A First Course.* // Prentice Hall. – 1975.