Nonlocal problem with integral conditions for homogeneous system of partial differential equations of second order

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Let $\Pi(T) = \{(t,x) \in \mathbb{R}^2 : t \in [0,T], x \in \mathbb{R}\}$, $T > 0$. Let us denote $E_{\alpha,\beta}, \alpha > 0, \beta > 0.$, to the space of functions $\varphi \in L_2(\mathbb{R})$, with the finite norm [1]

$$\|\varphi\|_{E_{\alpha,\beta}} = \sqrt{\frac{1}{2\pi}\int\limits_{-\infty}^{+\infty} |\hat{\varphi}(\xi)|^2 \left(1+|\xi|\right)^{2\alpha} \exp(2\beta|\xi|) d\xi}$$

where $\hat{\varphi}(\xi)$ is the Fourier transform of the function $\varphi(x)$. In the strip $\Pi(T)$ we consider nonlocal-integral problem

$$L\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}\right) u(t,x) \equiv \frac{\partial^n u(t.x)}{\partial t^n} + \sum_{j=1}^{n} a_j \frac{\partial^n u(t,x)}{\partial t^{n-j}\partial x^j} = 0, \quad a_j \in \mathbb{C}, \quad (t,x) \in \Pi(T), \tag{1}$$

$$\left.\frac{\partial^k U}{\partial t}\right|_{t=0} - \left.\frac{\partial^k U}{\partial t}\right|_{t=T} + \int\limits_0^T t^k U(t,x) dt = \varphi_k(x), \quad k = 0, ..., n-2, \tag{2}$$

$$\int\limits_0^{T_1} t^{n-1} U(t,x) dt + \int\limits_{T_2}^{T} t^{n-1} U(t,x) dt = \varphi_{n-1}(x) \tag{3}$$

where $a_1, a_2 \in \mathbb{C}$. Assuming that the real parts of roots of polynomial $\lambda^n + a_1\lambda^{n-1} + ... + a_n$ are nonzero and different, the correctioness of the problem (1) - (3) in the space of functions $C^2([0,T], E_{\alpha,\beta})$ is established.

Obtained results continue the research of the work [2].

References

[1] Yu. A. Dubinskii The algebra of pseudodifferential operators with analytic symbols and its applications to mathematical physicals. *Russian Mathematical Surveys*, (1982), 37(5): 109-153.

[2] P.I. Kalenyuk, Z.N. Nytrebych, M.M. Symotyuk, G, Kuduk. *Integral problem for partial differential equation of second order in unboudud layer*, *Bukovinian Mathematical Journal.* Vol. 4, No 3-4. (2016) - Chernivtsi: Chernivtsi Nat. Univ., - P. 69 - 74.