CANONICAL INFINITESIMAL DEFORMATIONS OF METRICS OF PSEUDO-RIEMANNIAN SPACES

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Difference of metric tensors of two pseudo-Riemannian spaces is called their deformation. Let V_n — be a pseudo-Riemannian space with a metric tensor g_{ij} , and \bar{V}_n — a pseudo-Riemannian space with a metric tensor \bar{g}_{ij} . Let us suppose that metric tensors differ by an infinitesimal small number γ_{ij} , or

$$\bar{g}_{ij} = g_{ij} + \gamma_{ij}.\tag{1}$$

Infinitesimal small numbers with an order above the first one will be discarded. Then, the following expression is true for tensors that are reversed in respect to metric tensors.

$$\bar{g}^{ij} = g^{ij} - g^{i\alpha}g^{j\beta}\gamma_{\alpha\beta}.$$

Components of tensor γ_{ij} are called components of the tensor field of velocities of infinitesimal small deformation.

While calculating other inner geometric objects, there is often a need to discard certain parameters. This way leads to the research on infinitesimal deformations of a metric. In this sense, infinitesimal parameters are parameters, which can be discarded not affecting the completeness of the problem under study.

Infinitesimally small deformation of type (1) of pseudo-Riemannian space (V_n, g_{ij}) is called canonical deformation when deformation tensor δg_{ij} can be represented in a shape

$$\gamma_{ij} = \tau^1 g_{ij} + \tau^2 R_{ij},$$

where $\overset{1}{\tau}, \overset{2}{\tau}$ — are some invariants [1, 2].

Since Saint-Venant's times, the deformation research is reduced to analysis of a system of differential equations. Saint-Venant's equations are the main tool for research on infinitesimal deformations. Saint-Venant's equations are understood here as a set of equations defining the deformation tensor in such a way that the space remains an Euclidean space.

Generalized Saint-Venant's equations are conditions under which Riemann tensor is preserved under infinitesimal deformations of a metric tensor of a pseudo-Riemannian space. They are differential equations in covariant derivatives in respect to tensors of Ricci and Riemann.

Conditions, which are imposed on tensors used for research on infinitesimal deformations, are both algebraic and differential. Having carried out the research of this type we are able to answer the question: whether the Saint-Venant's equations are true under the pre-defined conditions.

The research is carried out locally in tensor form, without limitations on a sign of a metric tensor.

References

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