Quasiareal deformation of surfaces of positive Gauss curvature

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In this paper it is considered quasiareal deformation of surfaces, which we will call also briefly QA-deformation. Quasiareal deformation is understood as an infinitesimal deformation of the first order with the given law of changing the element of area of a surface in Euclidean three-space.

Let $\overline{U}\left(x^1, x^2\right)$ be a field of velocities of the points of the surface $\overline{r} = \overline{r}\left(x^1, x^2\right)$ at the initial moment of the deformation, such that $\overline{U} = U^{\alpha}\overline{r}_{\alpha} + U^0\overline{n}$, where $\overline{r_i}, \overline{n}$, $i = 1, 2$, are the basis vectors. The fundamental equations of the quasiareal infinitesimal deformation, which are expressed in terms of the components of the partial derivatives of the field $\overline{U}$, are derived in [2].

It has been established: in order that the field $\overline{U} \in C^1$ be a deforming field of the quasiareal infinitesimal deformation it is necessary and sufficient that the components U^{α}, U^0 satisfy the equation

$$U^{\alpha}_{,\alpha} - 2HU^0 = -2\mu, \tag{1}$$

where the function μ expresses the law of changing the element of area.

It is evident, that the class of the QA-deformation is very wide since one differential equation (1) contains four unknown functions. It is expedient to study such deformation under the additional geometrical or mechanical conditions. For example, for the surface of positive Gauss curvature ($K > 0$) on the condition that $\delta\overline{n} = 0$ under the quasiareal infinitesimal deformation we have additional elliptic partial differential equation of the second order with respect to the normal component of the deforming field

$$d^{\alpha\beta}U^0_{\alpha,\beta} - \frac{K_{\alpha}}{K} d^{\alpha\beta}U^0_{\beta} + 2HU^0 = 2\mu. \tag{2}$$

The Riemann domain T has been described, in which the regular solution of the equation (2) exists for the regular surfaces of positive Gauss curvature, this solution is a continuous, non-zero everywhere in closed domain $\overline{T}$. This condition is a sufficient sign of the existence and uniqueness of the solution of the Dirichlet problem for the equation (2) [1].

The corresponding theorems have been formulated for the QA-deformation of the surfaces of positive Gauss curvature. QA-deformation in class of surfaces of constant mean curvature is discussed, for example, in a paper [3] and deformations preserving Gauss curvature in a paper [4].

References

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