# On the Carleman function for matrix factorizations of the Helmholtz equation

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This article deals with the construction of the Carleman function for matrix factorizations of the Helmholtz equation in a multidimensional domain.

It is known that the Cauchy problem for elliptic equations is unstable relatively small change in the data, i.e., is incorrect (Hadamard's example). In unstable problems the image of the operator is not closed, therefore the solvability condition can not be written in terms of continuous linear functionals. Thus, in the Cauchy problem for elliptic equations with data on a part of the boundary of the region, the solution is usually unique, the problem is solvable for an everywhere dense set of data, but this set not closed. Consequently, the theory of solvability of such problems is essentially It is more difficult and deeper than the theory of solvability of the Fredholm equations. The first results in this direction appeared only in the mid-1980s in the works of L.A. Aizenberg, A.M. Kytmanov, N.N. Tarkhanov (See, for instance [1]).

Let $x = (x_1, ..., x_m), y = (y_1, ..., y_m)$ be are points of the Euclidean space $\mathbb{R}^m$ and $G \subset \mathbb{R}^m$ be a bounded simply-connected domain with piecewise smooth boundary consisting of the plane $T$: $y_m = 0$ and of a smooth surface $S$ lying in the half-space $y_m > 0$, that i.e., $\partial G = S \bigcup T$.

We consider in the domain $G$ a system of differential equations

$$D\left(\frac{\partial}{\partial x}\right) U(x) = 0, \tag{1}$$

where $D\left(\frac{\partial}{\partial x}\right)$ is the matrix of first-order differential operators.

We denote by $A(G)$ the class of vector functions in a domain $G$ continuous on $\overline{G} = G \bigcup \partial G$ and satisfying system (1).

We define the function $\Phi(y, x; \lambda)$ at $y \neq x$ by the following equalities:

$$\Phi(y, x; \lambda) = \frac{1}{c_m K(x_m)} \frac{\partial^{k-1}}{\partial s^{k-1}} \int\limits_0^\infty \operatorname{Im}\left[\frac{K(w)}{w - x_m}\right] \frac{u I_0(\lambda u)}{\sqrt{u^2 + \alpha^2}} du, \; m = 2k, \; k \geq 1, \tag{2}$$

$$\Phi(y, x; \lambda) = \frac{1}{c_m K(x_m)} \frac{\partial^{k-1}}{\partial s^{k-1}} \int\limits_0^\infty \operatorname{Im}\left[\frac{K(w)}{w - x_m}\right] \frac{\cos(\lambda u)}{\sqrt{u^2 + \alpha^2}} du, \; m = 2k+1, \; k \geq 1, \tag{3}$$

where

at $m = 2k$, $k \geq 1$; $c_2 = 2\pi$, $c_m = (-1)^k 2^{-k} (m-2)\pi\omega_m (k-1)!$, $I_0(\lambda u) = J_0(i\lambda u)$–is the Bessel function of the first kind of zero order;

at $m = 2k+1$, $k \geq 1$; $c_m = (-1)^k 2^{-k} (2k-1)!(m-2)\pi\omega_m$, $\omega_m$– the area of a unit sphere in space $\mathbb{R}^m$.

In the future, using formulas (2) and (3), we will construct the Carleman matrix for matrix factorizations of the Helmholtz equation in multidimensional bounded domain and based on it we will find an approximate solution to the Cauchy problem in explicit form, using the methodology of previous works (See, for instance [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17] and [18]).

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