SPECIAL MEAN AND TOTAL CURVATURE OF A DUAL SURFACE IN ISOTROPIC SPACES

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In this paper, we study the properties of the total and mean curvatures of a surface and its dual image in an isotropic space. We prove the equality of the mean curvature and the second quadratic forms. The relation of the mean curvature of a surface to its dual surface is found. The superimposed space method is used to investigate the geometric characteristics of a surface relative to the normal and special normal.

Consider an affine space A_3 with the coordinate system $Oxyz$. Let $\vec{X}(x_1, y_1, z_1)$ and $\vec{Y}(x_2, y_2, z_2)$ be vectors of A_3.

Definition 1. If the scalar product of the vectors $\vec{X}$and $\vec{Y}$ is defined by the formula

$$\begin{cases} (X,Y)_1 = x_1x_2 + y_1y_2 & \text{if} \quad x_1x_2 + y_1y_2 \neq 0, \\ (X,Y)_2 = z_1z_2 & \text{if} \quad x_1x_2 + y_1y_2 = 0, \end{cases} \tag{1}$$

then A_3 is said to be an isotropic space R_3^2. [1, 2]

Geometry in a plane of an isotropic space will be Euclidean if it is not parallel to the oz axis. When a plane is parallel to oz, the geometry on it will be Galilean.

Since an isotropic space has an affine structure, there is an affine transformation that preserves the scalar product by formula (1). This motion of an isotropic space is given by the formula [5]

$$\begin{cases} x' = x\cos\alpha - y\sin\alpha + a \\ y' = x\sin\alpha + y\cos\alpha + b \\ z' = Ax + By + z + c \end{cases} \tag{2}$$

The second sphere is defined as a surface with the constant normal curvature. This sphere of the unit radius has the equation [8]

$$x^2 + y^2 = 2z, \tag{3}$$

we call it the isotropic sphere.

Let a plane π be given in R_3^2, which is not parallel to the oz axis of the space. Consider the section of the isotropic sphere by the plane π and denote it by Γ. Since an isotropic sphere is a paraboloid of revolution, the section Γ by a plane is a closed curve. It was proved in [2] that Γ is an ellipse.

Draw tangent planes to isotropic sphere (3) through points$M \in \Gamma$. Denote the set of tangent planes to points F by $\{\pi\}$.

The following statement holds.

Theorem 2. *All planes of the set $\{\pi\}$ intersect at one point.* [6]

If a plane π_0 is given by the equation

$$z = Ax + By + C, \tag{4}$$

then the intersection point of the planes of the set $\{\pi\}$ will be $(A, B, -C)$.

Definition 3. The point $(A, B, -C)$ will be called dual to plane (4) with respect to isotropic sphere (3). [6]

Let us draw the tangent plane π_M to the surface F at the point $M(x_0, y_0, z_0)$. Denote by M^* the dual image of the tangent space π_M with respect to the isotropic sphere. When the point $M \in F$ changes on the surface F, its dual image describes a surface F^*.

Definition 4. . The surface F^* is said to be the dual surface to the surface F in an isotropic space. [6]

When F is given by the equation $z = f(x, y)$, F^* has the equations

$$\begin{cases} x^*(u,v) = f'_u(u,v) \\ y^*(u,v) = f'_v(u,v) \\ z^*(u,v) = u \cdot f'_u(u,v) + v \cdot f'_u(u,v) - f(u,v) \end{cases} \tag{5}$$

Lemma 5. *When the total curvature of a surface $K = 0$, its dual image is a point or a curve.*

Theorem 6. *The product of the total curvatures of the surface $F$and the dual surface F^* of the isotropic space is equal to unity:*

$$K \cdot K^* = 1. \tag{6}$$

Lemma 7. *The special mean curvatures of the surfaces, given by the functions $\overrightarrow{R_1}(u,v) = f_u \cdot \overrightarrow{i} + f_v \cdot \overrightarrow{j} + f_u \cdot \overrightarrow{k}$ and $\overrightarrow{R_2}(u,v) = f_u \cdot \overrightarrow{i} + f_v \cdot \overrightarrow{j} + f_v \cdot \overrightarrow{k}$, are calculated, respectively, by the formulas*

$$H_m(R_1) = \frac{f_{uvv}\left(f_{uu}^2 + f_{uv}^2\right) - 2f_{uuv}\left(f_{uu}f_{uv} + f_{uv}f_{vv}\right) + f_{uuu}\left(f_{uv}^2 + f_{vv}^2\right)}{\left[f''_{uu}f''_{vv} - f''_{uv}{}^2\right]^2}, \tag{7}$$

$$H_m(R_2) = \frac{f_{vvv}\left(f_{uu}^2 + f_{uv}^2\right) - 2f_{uvv}\left(f_{uu}f_{uv} + f_{uv}f_{vv}\right) + f_{uuv}\left(f_{uv}^2 + f_{vv}^2\right)}{\left[f''_{uu}f''_{vv} - f''_{uv}{}^2\right]^2}. \tag{8}$$

Lemma 8. *The mean curvatures of the surfaces, given by the functions $\overrightarrow{R_1}(u,v)$ and $\overrightarrow{R_2}(u,v)$, are equal to zero.*

Lemma 9. *The mean curvature and special mean curvature of the dual surface* (5) *and the surfaces $R_1(u,v)$, $R_2(u,v)$ are connected by the equality:*

$$H_m^* = H^* + u \cdot H_m(R_1) + v \cdot H_m(R_2). \tag{9}$$

Theorem 10. *The mean curvatures defined with respect to the normal and the special normal are equal: $H_m^* = H^*$.*

Theorem 11. *If $\Omega = 0$, then the special total curvature of the surface F^* is expressed in terms of the special total curvatures of the surfaces F, Z_1, and Z_2.*

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