

THE COLLATZ CONJECTURE FROM AN ALGEBRAIC POINT OF VIEW

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The Collatz conjecture is an open problem in number theory established in 1937 by Lothar Collatz and can be stated as follows: If $f : \mathbb{N} \rightarrow \mathbb{N}$ is the function defined by:

$$f(n) = \begin{cases} \frac{n}{2} & ; n \text{ is even} \\ 3n + 1 & ; n \text{ is odd} \end{cases}$$

the conjecture says that given $n \in \mathbb{N}$, there exists $k > 0$ such that $f^{(k)}(n) = 1$ and the only orbit is $\{1, 2, 4\}$

Every topology τ can be seen as a commutative semiring under union and intersection. If τ_f is the topology on \mathbb{N} given by the open sets as those subset θ of \mathbb{N} such that $f^{-1}(\theta) \subset \theta$, we prove that the Collatz conjecture is true if and only if τ_f , viewed as a commutative semiring, is a local semiring.

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