Monogenic functions with values in commutative complex algebras of the second rank with unity and generalized biharmonic equation with non-zero simple characteristics

Serhii Gryshchuk
(Institute of Mathematics of NAS of Ukraine, Kyiv, Ukraine)
E-mail: serhii.gryshchuk@gmail.com

Among all two-dimensional algebras of the second rank with unity e over the field of complex numbers $\mathbb{C}$, we found a semi-simple algebra $\mathbb{B}_0 := \{c_1 e + c_2\omega : c_k \in \mathbb{C}, k = 1, 2\}$, $\omega^2 = e$, containing bases $\{e_1, e_2\}$, such that $\mathbb{B}_0$-valued "analytic" functions $\Phi(xe_1 + ye_2)$ (x, y are real variables) satisfy the fourth order homogeneous partial differential equation of the form:

$$\left(b_1\frac{\partial^4}{\partial y^4} + b_2\frac{\partial^4}{\partial x\partial y^3} + b_3\frac{\partial^4}{\partial x^2\partial y^2} + b_4\frac{\partial^4}{\partial x^3\partial y} + b_5\frac{\partial^4}{\partial x^4}\right)u(x, y) = 0, \tag{1}$$

where complex coefficients $b_k \in \mathbb{C}$, $k = \overline{1,5}$, $b_5 \neq 0$, such than the Eq. of characteristics

$$l(s) := b_1 s^4 + b_2 s^3 + b_3 s^2 + b_4 s + b_5 = 0, s \in \mathbb{C}, \tag{2}$$

has four pairwise different roots (each root is a simple root).

A set of pairs $(\{e_1, e_2\}, \Phi)$, where all real components of Φ satisfy Eq. 1, is described in the explicit form.

A totalies of "analytic" functions $\Phi(xe_1 + ye_2)$, such that the first real component of each of them satisfies the given solution u of Eq. 1 in the simply-connected bounded domains, are found in [2, 3, 5, 6].

Particular cases of this research are considered in [1, 2, 3, 4, 5].

The complete statements, proofs and definitions are considered in [6].

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References

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