Metric viewpoint in mapping theory between Riemannian manifolds

Elena Afanas'eva
(Institute of Applied Mathematics and Mechanics of the NAS of Ukraine, 1 Dobrovol'skogo St., Slavyansk 84100, Ukraine)
E-mail: es.afanasjeva@gmail.com

Anatoly Golberg
(Department of Mathematics, Holon Institute of Technology, 52 Golomb St., P.O.B. 305, Holon 5810201, ISRAEL, Fax: +972-3-5026615)
E-mail: golberga@hit.ac.il

The theory of multidimensional quasiconformal mappings employs three main approaches: analytic, geometric (modulus) and metric ones. In this talk, we use the last approach and establish the relationship between various classes of mappings on Riemannian manifolds including homeomorphisms of finite metric distortion (FMD-homeomorphisms), finitely bi-Lipschitz, quasisymmetric and quasiconformal mappings. The appropriate classes of homeomorphisms involving the modulus technique are also presented. One of the main results shows that FMD-homeomorphisms are lower Q-homeomorphisms. As an application, there are obtained some sufficient conditions for boundary extensions of FMD-homeomorphisms. These conditions are illustrated by several examples of FMD-homeomorphisms.

A classical example of significance of metric approach can be illustrated by the Bohr-Menchoff-Trokhymchuk theory on analyticity (monogeneity) of a complex variable function. In 1937 Menchoff [2] generalized the Bohr theorem [1] on analytic functions in the terms of preserving infinitesimal circles. More precisely, for a continuous and locally univalent mapping $w = f(z)$ of a domain D onto a domain $D*$ and $z_0 \in D$, take the quantity

$$H(z_0, r) = \frac{\max\limits_{|z'-z_0|=r} |f(z') - f(z_0)|}{\min\limits_{|z''-z_0|=r} |f(z'') - f(z_0)|}$$

and say that f preserves infinitesimal circles in D if $H(z_0, r) \to 1$ as $r \to 0$. The Menchoff result states that the preserving infinitesimal circles at all z_0 except for at most a countable set completely provides that either f or its conjugate is analytic in D. This pure metric condition has been extended to continuous mappings by Yu. Yu. Trokhymchuk [3] involving the Stoilow theory on interior mappings.

The classes of mappings presented in the talk can be treated as far advanced extensions of the Bohr-Menchoff-Trokhymchuk theory on complex plane to more general structures.

References

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