On Orthosymmetric n-morphisms

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Let E and F be vector lattices.We say that a bilinear mapping $T : E \times E \to F$ is an orthosymmetric mapping if $T(x, y) = 0$ in F , whenever $\mid x \mid \wedge \mid y \mid= 0$ in E. Generalization of this definition for n linear mapping is that $T : E \times E \times ... \times E \to F$ is an orthosymmetric multilinear mapping if $T(x_1, x_2, ..., x_n) = 0$ for all $x_1, ..., x_n \in E$ such that $\mid x_i \mid \wedge \mid x_j \mid= 0$ for some pair of indices $1 \leq i, j \leq n$. By $E^{\sim}$ we denote the set of all order bounded linear functionals on E. $E_n^{\sim}$ denotes the set of all order continuous linear functionals on E. By $(E^{\sim})_n^{\sim}$ we denote the order continuous order bidual of E. Let $E_1, ..., E_n$ and F be vector lattices. A multilinear mapping $\Psi : E_1 \times ... \times E_n \to F$ is said to be a lattice n-morphism if $\mid \Psi(x_1, ..., x_n) \mid= \Psi(\mid x_1 \mid, ..., \mid x_n \mid)$ for all $x_i \in E_i$ for $i = 1, 2, ..., n$. We say that a lattice n-morphism and orthosymmetric multilinear mapping is an orthosymmetric n-morphism.

Orthosymmetric bilinear mappings have been studied by a lot of authors. For example, M.A. Toumi and R. Yilmaz give the extensions of orthosymmetric bilinear mapping to the order continuous order bidual of a vector lattice by using Arens multiplication.

In this study, we extend an orthosymmetric n-morphism to the order continuous order bidual of a vector lattice by using Arens product. We show that an extension of orthosymmetric n-morphism is again orthosymmetric n-morphism. Unexplained notion and terminology we refer to the following references.

Theorem 1. *Suppose that E is an Archimedean vector lattice and F is a Dedekind complete vector lattice. If $\Psi : E \times E \times ... \times E \to F$ is an orthosymmetric n-morphism, then n-th order adjoint of Ψ on the order continuous order bidual $(E^{\sim})_n^{\sim}$ of E is again an orthosymmetric n-morphism.*

References

[1] C.D. Aliprantis, O. Burkinshaw. *Positive Operators*Academic Press, London, 1985.
[2] R. Arens. The adjoint of a bilinear operation. *Proc. Am. Math. Soc.*, 2, 839-848, 1951.
[3] R. arens. Operations induced in function classes. *Monatsh. fur Math.*, 55,1-19, 1951.
[4] Z. Kusraeva. Sums of disjointness preserving multilinear operators. *Positivity*, 2020.
[5] M. A. Toumi. The triadjoint of an orthosymmetric bimorphism. *Czechoslovak Math. J.*, 60, 85-94, 2010.
[6] R.Yilmaz. The Arens triadjoints of some bilinear maps. *Filomat*, 28, 963-979, 2014.