On symmetry reduction and some classes of invariant solutions of the $(1+3)$-dimensional homogeneous Monge-Ampère equation

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A solution of many problems of the geometry, theoretical and mathematical physics has reduced to the investigation of the Monge-Ampère equations in the spaces of different dimensions and different types.

It is well known that the symmetry reduction is one of the most powerful tools to investigate partial differential equations (PDEs) with non-trivial symmetry groups. In particular, for this purpose, we can use a classical Lie-Ovsiannikov method. This method, among the other, makes it possible to perform the symmetry reduction and construction of invariant solutions of those equations.

In 1984, Grundland, Harnad, and Winternitz pointed out that the reduced equations, obtained with the help of nonconjugate subalgebras of the same ranks of the Lie algebras of the symmetry groups of some PDEs, were of different types. They also investigated the similar phenomenon. The results obtained cannot be explained using the classical Lie-Ovsiannikov approach.

To try to explain some of the differences in the properties of the reduced equations for PDEs with nontrivial symmetry groups, we suggested to investigate the relationship between the structural properties of nonconjugate subalgebras of the same rank of the Lie algebras of the symmetry groups of those PDEs and the properties of the reduced equations corresponding with them.

At the present time, we have investigated the relationship between structural properties of the three-dimensional nonconjugate subalgebras of the same rank of the Lie algebra of the Poincaré group $P(1,4)$ and the properties of reduced equations for the (1+3)-dimensional homogeneous Monge-Ampére equation. We obtained the following types of the reduced equations:
- identities,
- the linear ordinary differential equations,
- the nonlinear ordinary differential equations,
- the partial differential equations.

Some classes of invariant solutions have been constructed.

In my report, I plan to present some of the results obtained concerning with reduction of the (1+3)-dimensional homogeneous Monge-Ampére equation to identities.

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