Parallel spinors on Lorentzian Weyl spaces

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In that talk I will present the recent results of joint work with Anton S. Galaev on Lorentzian Weyl spin manifolds admitting weighted parallel spinors [2].

Parallel spinors are special Killing spinors which represent supersymmetry generators of supersymmetric field theories and supergravity theories. The physical motivation for study Weyl spaces with weighted parallel spinors may be found in [3]. The work [3] provides a deep investigation of weighted parallel spinors on Einstein-Weyl manifolds of Lorentzian signature with a special attention to the dimensions 4 and 6. The techniques developed for classification of supergravity solutions was used in that work.

We provide a description of simply connected Lorentzian Weyl spin manifolds admitting weighted parallel spinors. The main tool for that are holonomy groups. There is a one-to-one correspondence between the parallel spinors and the holonomy-invariant elements of the spinor module. This correspondence is used in known results describing the following simply connected spin manifolds with parallel spinors: Riemannian manifolds, pseudo-Riemannian manifolds with irreducible holonomy groups, Lorentzian manifolds.

Using the recent classification of holonomy algebras of Lorentzian Weyl manifolds [1], we classify the holonomy algebras of Lorentzian Weyl spaces admitting weighted parallel spinors. It turns out that for non-closed Weyl structures, there are two types of such algebras. In each case, the dimension of the space of parallel spinors is found.

For Lorentzian Weyl manifolds admitting recurrent null vector fields are introduced special local coordinates similar to Kundt and Walker ones. Using that, the local form of all Lorentzian Weyl spin manifolds with weighted parallel spinors is given. The Einstein-Weyl equation for the obtained Weyl structures is analyzed and examples of Einstein-Weyl spaces with weighted parallel spinors are given. Some examples have previously appeared in [3] and other literature. It turns out that the Einstein-Weyl equation implies that the weight of a non-zero weighted parallel spinor is equal to $\dim M - 4$. Parallel spinors of that weight were studied in [3]. In contrast, we describe Weyl structures with non-zero weighted parallel spinors of arbitrary weight.

References

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