# Generalized $(\sigma,\, \tau)$-Derivations on Associative Rings Satisfying Certain Identities

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The commutativity of associative rings with derivations have become one of the focus points of several authors and a significant work has been done in this direction during the last two decades. It represents the answer to the natural questions of Ring Theory which reach to determine the conditions implying commutativity of the ring. Basically, the study of derivation was initiated during the 1950s and 1960s. Derivations of rings got a tremendous development in 1957, when Posner [1] established two very striking results in the case of prime rings. A considerable amount of work has been done on derivations and related maps during the last decades (see, e.g., [2,3 and 4] and references therein).The main purpose of this paper is present results concerning generalized $(\sigma,\, \tau)$-derivations via associative rings. Accurately, we prove the commutativity with other cases of a ring that satisfied certain conditions. These results are in the sprite of the well-known theorem of the commutativity of prime and semiprime rings with generalized derivation satisfying certain polynomial constraints. Throughout this paper, $R$ always represents an associative ring and $Z(R)$ is its center. Let $\sigma$ and $\tau$ be two mappings from $R$ to itself. For any $x, y \in R$ we write $[x, y]_{(\sigma,\tau)}$ for the commutator $x\sigma(y) - \tau(y)x$ and $(x \circ y)_{(\sigma,\tau)}$ for anti-commutator $x\sigma(y) + \tau(y)x$.
Recall that $R$ is semiprime if $aRa = 0$ implies $a = 0$ and $R$ is prime if $aRb = 0$ implies $a = 0$ or $b = 0$. Every prime ring is semiprime ring but the converse is not true always. An additive mapping $d : R \longrightarrow R$ is said to be an $(\sigma,\, \tau)$-derivation of $R$ if $d(xy) = d(x)\sigma(y) + \tau(x)d(y)$ holds for $x, y \in R$. Let $\sigma$ and $\tau$ be endomorphisms of $R$. An additive mapping $D : R \longrightarrow R$ is said to be a generalized $(\sigma,\, \tau)$-derivation of $R$ if there exists an $(\sigma,\, \tau)$-derivation $d : R \longrightarrow R$ of $R$ such that $D(xy) = D(x)\sigma(y) + \tau(x)d(y)$ for all $x, y \in R$.

**Theorem 1.** *Let $R$ be a non-zero semiprime ring with nonzero commutator, $\sigma$ and $\tau$ be automorphsim mappings. If $R$ admits a generalized $(\sigma,\, \tau)$-derivation satifises the identity $D(x)oy = D(xy)$ for all $x, y \in R$, then $D = 0$.*

**Theorem 2.** *Let $R$ be a 2-torsion free semiprime ring with nonzero commutator, $\sigma$ and $\tau$ be automorphsim mappings. If $R$ admits a generalized $(\sigma,\, \tau)$-derivation satisfies the identity $D(xoy) = D(x)oy - D(y)ox$ for all $x, y \in R$, then $d = 0$.*

## References

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