

ASYMPTOTIC ANALYSIS OF QUASI-REGULAR MAPPINGS IN SPACE

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Dedicated to the memory of Yuri Yu. Trokhymchuk

We investigate quasiregular embeddings of a closed 3-ball inextensible in neighborhoods of any boundary points and bounded quasiregular locally homeomorphic mappings in 3-space, especially their behavior in the unit 3-ball and their asymptotics while approaching the boundary of the unit 3-ball (cf. [12], [14], [15], [16], [17]).

We discover several new properties of such mappings in 3-space. Such discoveries are based on our construction of a new type of bounded locally homeomorphic quasiregular mappings in 3-sphere (and in the unit 3-ball)- see [6]. It addresses long standing problems for such mappings, including M.A.Lavrentiev problem, Pierre Fatou problem and Matti Vuorinen injectivity and asymptotics problems (cf. [7]). The construction of such mappings comes from our construction of non-trivial compact 4-dimensional cobordisms M with symmetric boundary components and whose interiors have complete 4-dimensional real hyperbolic structures (cf. [4]). Such bounded locally homeomorphic quasiregular mappings are defined in the unit 3-ball $B^3 \subset \mathbb{R}^3$ as mappings equivariant with the standard conformal action of uniform hyperbolic lattices $\Gamma \subset \text{Isom } H^3$ in the unit 3-ball and with its discrete representation $G = \rho(\Gamma) \subset \text{Isom } H^4$ (cf. [6]). Here G is the fundamental group of our non-trivial hyperbolic 4-cobordism $M = (H^4 \cup \Omega(G))/G$ and the kernel of the discrete representation $\rho: \Gamma \rightarrow G$ could be a free group F_m on arbitrary large number m generators.

Such discrete non-faithful representations of hyperbolic lattices with arbitrarily large kernel were known only for non-uniform case due to the W.Thurston's non-rigidity theorem (Dehn surgeries on cusp ends of non-compact hyperbolic 3-manifolds). We are able to present our construction for uniform (co-compact) hyperbolic 3-lattices based on a new effect in the theory of deformations of hyperbolic 3-manifolds/orbifolds or their uniform hyperbolic lattices $\Gamma \subset \text{Isom } H^3$ (i.e. in the Teichmüller spaces of conformally flat structures on closed hyperbolic 3-manifolds -cf. [1, 2]). We show that such Teichmüller space or the corresponding variety of conjugacy classes of discrete representations $\rho: \Gamma \rightarrow \text{Isom } H^4$ may have connected components whose dimensions differ by arbitrary large numbers -cf. [3, 5]. This is based on our enhancement to the conformal category of the Gromov-Piatetski-Shapiro interbreeding construction [13] and our construction of non-trivial "symmetric hyperbolic 4-cobordisms" [8].

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