About longest and shortest chords passing through a fixed point

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A new method to construct a tangent to the conchoid of Nicomedes or limaçon of Pascal curves is discussed. Some interesting properties of the cardioid curve (which is a special case of limaçon of Pascal) are investigated. The following problem is studied: "Given a line k and two points A and B on one side of k, find point C such that the sum of lengths of segments CD and CE is minimal, where D and E are intersections of line k with lines CA and CB, respectively". This problem is dual to the classic problem to find shortest segment inscribed to a given angle and passing through a given point. Part of this problem was solved and the remaining part is left as an open question. The problem to find ellipse's longest or shortest chord passing through a given point, is also considered. For the solution the curve named as ophiuride is used.

The following Lemma is used.

Lemma 1. *Let c_1 and c_2 be two arbitrary smooth curves. Let O be a given point and let a line through this point intersect the curves c_1 and c_2 at points A and B. If the length of segment AB is maximal/minimal or constant and the tangents to the curves c_1 and c_2 at points A and B are not perpendicular or parallel to the line AB then these tangents intersect at a point C such that for the perpendicular CD of the line AB the equality $|OA| = |BD|$ holds true.*

References

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