Minimal generating set and structure of a wreath product of groups and the fundamental group of an orbit of Morse function

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The quotient group of the restricted and unrestricted wreath product by its commutator is found. The generic sets of commutator of wreath product were investigated.

We generalize the results presented in the book of Meldrum J. [1] about commutator subgroup of wreath products since, as well as considering regular wreath products, we consider those which are not regular (in the sense that the active group $\mathcal{A}$ does not have to act faithfully). The fundamental group of orbits of a Morse function $f: M \to \mathbb{R}$ defined upon a Möbius band M with respect to the right action of the group of diffeomorphisms $\mathcal{D}(M)$ has been investigated.

Denote the set of all the orbits of $\mathcal{A}$ on X by $\mathcal{O}$, if this set is finite then by $\mathcal{O}_f$. Recall that the direct product indexed by infinite set consists of all infinite sequences, and the direct sum consists only of sequences with finitely many elements distinct from zero. Denote by $Z(\tilde{\triangle}(\mathcal{B}))$ the subgroup of diagonal subgroup [2] $Fun(X, Z(B))$ of functions $f : X \to Z(B)$ which are constant on each orbit of action of A on X for unrestricted wreath product, and denote by $Z(\triangle(\mathcal{B}^n))$ the subgroup of diagonal $Fun(X, Z(\mathcal{B}^n))$ of functions with the same property for restricted wreath product, where n is number of non-trivial coordinates in base of wreath product.

Theorem 1. *A centre of the group $(\mathcal{A}, X) \wr \mathcal{B}$ is direct product of normal closure of centre of a diagonal of $Z(\mathcal{B}^n)$ i.e. $(E \times Z(\triangle(\mathcal{B}^n)))$, trivial an element, and intersection of $(\mathcal{K}) \times E$ with $Z(\mathcal{A})$. In other words,*

$$Z((\mathcal{A}, X) \wr \mathcal{B}) = \langle (1;\ \underbrace{h, h, \ldots, h}_{n}),\ e,\ Z(\mathcal{K}, X) \wr \mathcal{E} \rangle \simeq (Z(\mathcal{A}) \cap \mathcal{K}) \times Z(\triangle(\mathcal{B}^n)),$$

where $h \in Z(\mathcal{B})$, $|X| = n$.

For restricted wreath product with n non-trivial coordinate: $Z((\mathcal{A}, X) \wr \mathcal{B}) =$
$\langle (1;\ \ldots, h, h, \ldots, h, \ldots),\ e,\ Z(\mathcal{K}, X) \wr \mathcal{E} \rangle \simeq (Z(\mathcal{A}) \cap \mathcal{K}) \times Z(\triangle(\mathcal{B}^n)) \simeq$
$\simeq \bigoplus\limits_{j \in O_f} (Z(\mathcal{A}) \cap \mathcal{K}) \times Z(\mathcal{B})$.

In case of unrestricted wreath product we have: $Z((\mathcal{A}, X) \wr \mathcal{B}) =$
$\langle (1;\ \ldots, h_{-1}, h_0, h_1, \ldots, h_i, h_{i+1}, \ldots,),\ e,\ Z(\mathcal{K}, X) \wr \mathcal{E} \rangle \simeq (Z(\mathcal{A}) \cap \mathcal{K}) \times Z(\tilde{\triangle}(\mathcal{B}))$
$= \prod\limits_{j \in O} (Z(\mathcal{A}) \cap \mathcal{K}) \times Z(\mathcal{B})$.

Theorem 2. *If $W = (\mathcal{A}, X) \wr (\mathcal{B}, Y)$, where $|X| = n$, $|Y| = m$ and active group $\mathcal{A}$ acts on X transitively, then*

$$d\left(G'\right) \leq (n-1)d(\mathcal{B}) + d(\mathcal{B}') + d(\mathcal{A}').$$

Theorem 3. *The quotient group of a restricted wreath products $G = Z \wr_X Z$ by a commutator subgroup is isomorphic to $\mathbb{Z} \times \mathbb{Z}$. In previous conditions if $G = A \wr_X B$ then, $G/G' = A/A' \times B/B'$. If $G = Z_n \wr Z_m$, where $(m,\ n) = 1$, then $d(G/G') = 1$. If $G = Z \wr Z$ is an unrestricted regular wreath product then $G/G' \simeq Z \times E \simeq Z$.*

References

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