# On boundary behavior by prime ends of solutions to Beltrami equations

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It is shown that each **homeomorphic** $W^{1,1}_{\rm loc}$ solution to the Beltrami equation is the so-called lower $Q$-homeomorphism with $Q(z) = K_\mu(z)$ where $K_\mu(z)$ is the dilatation quotient of this equation. It is developed on this basis, see e.g. [2], the theory of the boundary behavior of such solutions.

Let $D$ be a domain in the complex plane $\mathbb{C}$ and let $\mu : D \to \mathbb{C}$ be a measurable function with $|\mu(z)| < 1$ a.e. in $D$. The **Beltrami equation** is the equation of the form

$$f_{\overline{z}} \;=\; \mu(z) f_z \tag{1}$$

where $f_{\overline{z}} = \overline{\partial} f = (f_x + i f_y)/2$, $f_z = \partial f = (f_x - i f_y)/2$, $z = x + iy$, and $f_x$ and $f_y$ are partial derivatives of $f$ in $x$ and $y$, correspondingly. The function $\mu$ is called the **complex coefficient** and

$$K_\mu(z) \;=\; \frac{1 + |\mu(z)|}{1 - |\mu(z)|} \tag{2}$$

the **dilatation quotient** for the equation (1) that is **degenerate** if $\operatorname{ess\,sup} K_\mu(z) = \infty$.

In [2] and [3], we follow Caratheodory in the definition of the **prime ends** for bounded finitely connected domains in $\mathbb{C}$ and refer readers to Chapter 9 in [1]. In what follows, $\overline{D}_P$ denotes the completion of the domain $D$ by its prime ends with the the **topology of prime ends**, see Section 9.5 in [1]. Further, we assume that $K_\mu$ is extended by 0 outside of $D$.

**Theorem 1.** *Let $D$ and $D'$ be bounded finitely connected domains in $\mathbb{C}$ and let $f : D \to D'$ be a homeomorphic $W^{1,1}_{\rm loc}$ solution of the Beltrami equation (1) with*

$$\int\limits_0^{\delta(z_0)} \frac{dr}{||K_\mu||(z_0, r)} \;=\; \infty \qquad\qquad \forall\ z_0 \in \partial D \tag{3}$$

*where $0 < \delta(z_0) < d(z_0) = \sup\limits_{z \in D} |z - z_0|$ and $||K_\mu||(z_0, r) := \int\limits_{|z-z_0|=r} K_\mu(z)\, |dz|$. Then $f$ can be extended to a homeomorphism of $\overline{D}_P$ onto $\overline{D'}_P$.*

## References

[1] E.F. Collingwood, A.J. Lohwator. *The Theory of Cluster Sets*, volume 56 of *Cambridge Tracts in Math. and Math. Physics*. Cambridge: Cambridge Univ. Press, 1966.

[2] D. Kovtonyuk, I. Petkov, V. Ryazanov. Prime ends in theory of mappings with finite distortion in the plane. *Filomat*, volume 31 (5) : 1349–1366, 2017.

[3] I.V. Petkov. The boundary behavior of homeomorphisms of the class Wloc1,1 on a plane by prime ends. *Dopov. Nac. akad. nauk Ukr.*, 6 : 19–23, 2015 [in Russian]. https://doi.org/10.15407/dopovidi2015.06.019