The density and the $\tau-$placed of the $N_\tau^\varphi-$nucleus of a space X

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Let X be a topological space and let ξ be a system of closed subsets of X. The system ξ is called *linked*, if any two elements of it have nonempty intersection. A linked system ξ of closed subsets of X is called *complete*, if for any closed set $F \subset X$, the condition

" any neighborhood OF contains the set $\Phi \in \xi$ " $(\star)$

implies that $F \in \xi$ [1]. The set of all complete linked systems (CLS) in the space X is denoted by NX.

Let $U_1, \ldots, U_k, V_1, \ldots, V_n$ be a set of nonempty open subsets of X. Set $O(U_1, \ldots, U_k)\langle V_1, \ldots, V_n\rangle = \{\xi \in NX$: for any $i = 1, \ldots, k$ there exists $F_i \in \xi$ such that $F_i \subset U_i$, and for any $j = 1, \ldots, n$ and any $\Phi \in \xi$, the intersection $\Phi \cap V_j$ is nonempty $\}$. It is easily seen that the set of subsets of NX of the form $O(U_1, \ldots, U_k)\langle V_1, \ldots, V_n\rangle$ is an open basis of some topology on NX.

Definition 1. Let X be a T_1-space, φ be a cardinal-valued function, and τ be a cardinal number. The $N_\tau^\varphi-$nucleus of a space X is the space

$$N_\tau^\varphi X = \{\xi \in NX : \text{there exists } F \in \xi \text{ such that } \varphi(F) \leq \tau\} \text{ [2]}.$$

Definition 2. A topological space X is said to be $N_\tau^\varphi-$nuclear if $N_\tau^\varphi X = NX$.

As φ, we take a density function d. Let $\tau = \aleph_0$.

The definition implies that any space X is N_τ^d-nuclear, where $\tau = d(X)$; in particular, any separable space X is $N_{\aleph_0}^\varphi-$nuclear.

Theorem 3. *Let X be an infinite T_1-space. Then*
1) $\pi w(N_{\aleph_0}^d X) = \pi w(X)$;
2) $d(N_{\aleph_0}^d X) = d(X)$ (taken from [2]).

A set $A \subset X$ is called *$\tau-$placed* in X if for each point $x \in X \setminus A$ there is a set P of type G_τ in X such that $x \in P \subset X \setminus A$ [3].

Put $q(X) = min\{\tau \geq \aleph_0 : X$ is $\tau-$placed in $\beta X\}$; $q(X)$ is called the *Hewitt-Nachbin number* of X. We say that X is a *$Q_\tau-$space* if $q(X) \leq \tau$.

A space X is called an $m_\tau-$space, where τ is given cardinal, if for each canonical closed set F in X and each point $x \in F$ there is set P of type G_τ in X such that $x \in P \subset F$.

Clearly, X is an $m_\tau-$space for $\tau = |X|$. This allows us to give the following definition: put $m(X) = min\{\tau \geq \aleph_0 : X$ is an $m_\tau-$space $\}$. The space X is called a *Moscow space* if $m(X) \leq \aleph_0$.

Theorem 4. *Let $q(N_{\aleph_0}^d X) \leq \tau$ and $m(NX) \leq \tau$, then $N_{\aleph_0}^d X$ is $\tau-$placed in NX.*

Theorem 5. *Let $m(NX) \leq \tau = d(X)$, then an $N_{\aleph_0}^d-$nucleus $N_{\aleph_0}^d X$ is $\tau-$placed in NX.*

Theorem 6. *Let NX is a Moscow space and X is a separable, then $N_{\aleph_0}^d X$ is $\tau-$placed in NX.*

References

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